Compressed Sensing Crash Intro

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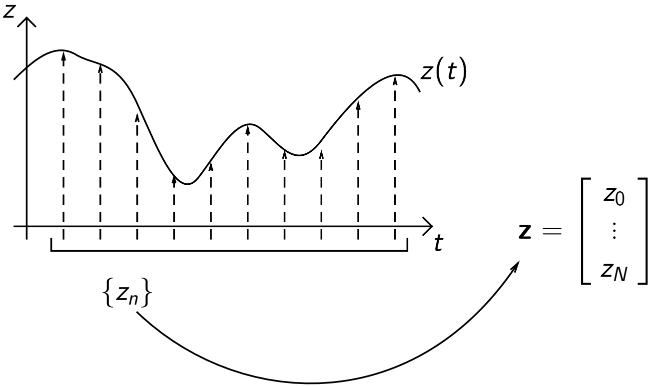
UC Louvain, Louvain-la-Neuve, November 30th, 2012

Traditional Nyquist Sampling

Classic Nyquist sampling:



$$\mathbf{z} = \begin{bmatrix} z_0 \\ \vdots \\ z_N \end{bmatrix}$$

Traditional Nyquist Sampling

Compression

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Compression

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- We sample $\mathbf{z}$.

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- *Then* compress; e.g. MP3, JPEG etc.

Traditional Nyquist Sampling

Compression

Classic Nyquist sampling:

- We sample $\mathbf{z}$.
- *Then* compress; e.g. MP3, JPEG etc.
- Conclusion: we sample a lot of data, but throw most of it away.

Compressed Sensing

Basics

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Basics

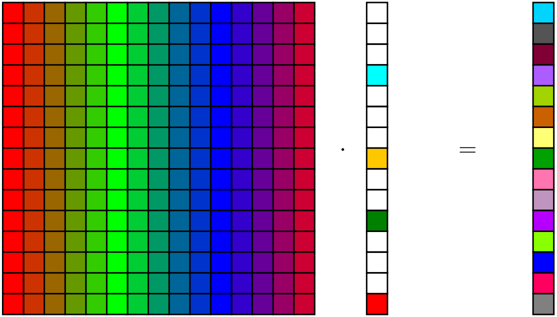
What is compressed sensing?

- New signal acquisition/compression theory from around 2004.
- Combines sampling and compression of signals.
- Interesting implication: Signals can be sampled significantly below Nyquist rate!

Compressed Sensing

Requirements

The signal must be sparse in a *known dictionary*:



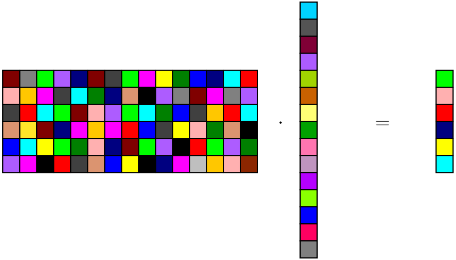
dictionary matrix $\mathbf{\Psi}$

sparse vector $\mathbf{x}$

original signal vector $\mathbf{z}$

Compressed Sensing

Acquisition

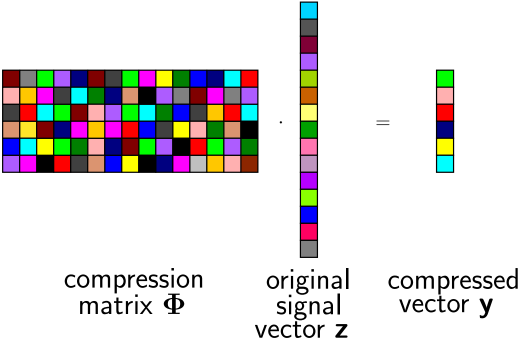


compression matrix $\mathbf{\Phi}$

original signal vector $\mathbf{z}$

compressed vector $\mathbf{y}$

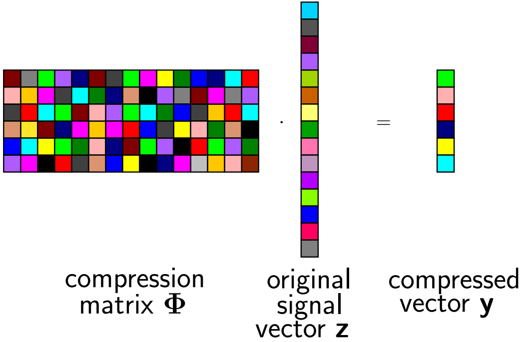
Compressed Sensing
Acquisition



- The signal vector is mixed with a *measurement* matrix before sampling.

Compressed Sensing

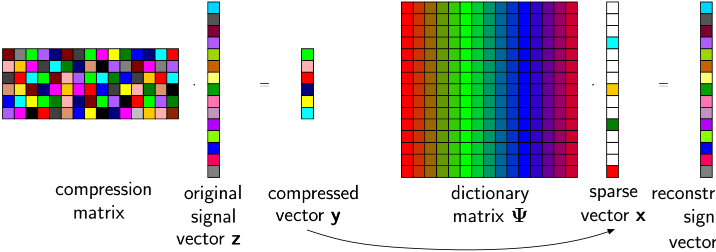
Acquisition



- The signal vector is mixed with a *measurement* matrix before sampling.
- Sample the (fewer) mixed “measurements”.

Compressed Sensing

The Reconstruction Principle



compression matrix · original signal vector **z** = compressed vector **y**

dictionary matrix **Ψ** · sparse vector **x** = reconstructed signal vector $\hat{\mathbf{z}}$

optimization-based reconstruction

Compressed Sensing
Conclusion

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Compressed Sensing

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When to use CS

Compressed Sensing

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  For example:

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  For example:
  - Sensor is expensive to build with sufficiently high resolution/rate.

Compressed Sensing

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  For example:
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  - Collecting enough samples takes too long.

Compressed Sensing
Conclusion

Is compressed sensing (CS) always a good idea?

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When to use CS

- When samples are otherwise too “expensive” to take...
  For example:
  - Sensor is expensive to build with sufficiently high resolution/rate.
  - Collecting enough samples takes too long.
  - Collecting fewer samples saves significant energy.

Compressed Sensing in RF Communication and Analog-to-Digital Conversion

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Agenda

Projects and People

RF Communication

Reconstruction

D/A Conversion

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Projects and People

Sparse Signal Processing in Wireless Communications (SparSig)

- New ways to sample, quantize, process and re-synthesize analog signals.
- Focus on wireless communication and signals for such a system.

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Projects and People

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- New ways to sample, quantize, process and re-synthesize analog signals.
- Focus on wireless communication and signals for such a system.
- Main contributions: to develop a framework for handling realistic signals (which are noisy, distorted etc.) and to provide a convincing validation (including experiments).
- Objective: to reduce the power consumption of converters and digital signal processing in devices typically used in wireless communication systems.

Projects and People

Sparse Signal Processing in Wireless Communications (SparSig)

Participants

- Prof. Torben Larsen
- Prof. Søren Holdt Jensen (MISP section)
- Assist. Prof. Thomas Arildsen
- Post doc. Tobias Lindstrøm Jensen (partly MISP)
- PhD stud. Karsten Fyhn (partly MISP)
- PhD stud. Peng Li
- PhD stud. Paweł Pankiewicz

The
Danish Council for
Strategic Research

Projects and People

4th Generation Mobile Communication and Test Platform (4GMCT)

The compressed sensing part of this project

- Wireless communication in 4G communication systems.
- Concepts and electronic circuits to perform energy-efficient sampling, quantization and processing of signals.

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Projects and People

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- Wireless communication in 4G communication systems.
- Concepts and electronic circuits to perform energy-efficient sampling, quantization and processing of signals.
- Purpose: investigate theoretical basis, analyze and propose solutions for implementing compressed sampling techniques in communication receivers.
- Main issue: to propose solutions for sub-Nyquist sampling and quantization, as well as a reconstruction algorithm design.

Projects and People

4th Generation Mobile Communication and Test Platform (4GMCT)

Participants (compr. sens. part)

- Prof. Torben Larsen
- PhD stud. Hao Shen
- PhD stud. Jacek Pierzchlewski

Projects and People

Compressive Sensing in Signal Analyzer

- Main purpose: to advance the compressed sensing theory and calculation process to enable utilizing compressed sensing in real-life applications.
- In particular signals analyzer equipment.

Participants

- Prof. Torben Larsen
- PhD stud. Ruben Grigoryan

Agenda

Projects and People

RF Communication

Reconstruction

D/A Conversion

RF Communication

Downsampling of DFT Precoded Signals for the AWGN Channel

- We connect so-called *k*-simple vectors to decoding of digital signals.
- Data vector *not* sparse: $\tilde{x} = [0\ 1\ 1\ 0\ 1\ 0\ 1\ 1 \ldots]^T$.
- We extend an existing linear programming recovery method with a semidefinite recovery method.

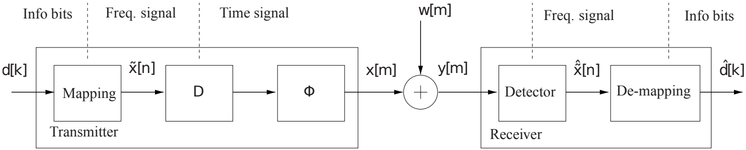
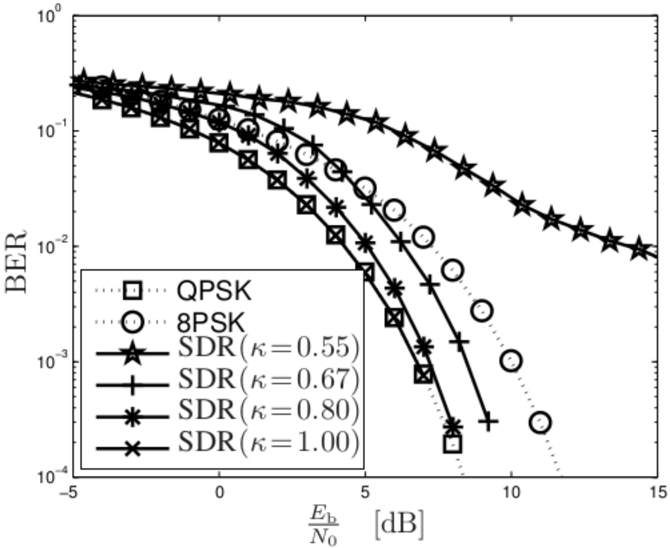


Figure: Encoder-decoder principle

RF Communication

Downsampling of DFT Precoded Signals for the AWGN Channel



RF Communication

Compressive Sensing for Spread Spectrum Signals

- Compressive sensing in a CDMA or DSSS receiver.
- Possibility to decrease the sampling rate.

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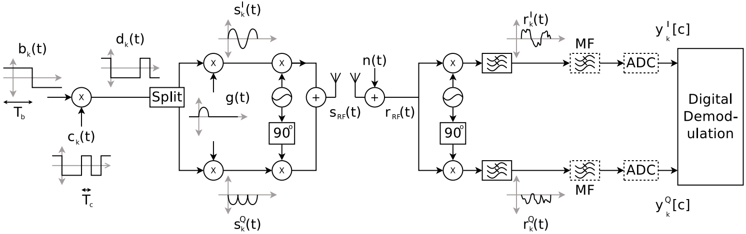
- Possibility to decrease the sampling rate.
- Made possible by the spreading codes used to better combat interference or low SNR at the receiver.
- These spreading codes decrease the information rate per chip or bit sent, which enables a sparse decomposition approach.

RF Communication

Compressive Sensing for Spread Spectrum Signals

- Example with IEEE 802.15.4 physical layer (for example ZigBee).
- Notice no random linear mixing necessary in receiver.
- No compressed sensing reconstruction – simpler compressive classification in stead.



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RF Communication

Compressive Sensing for Spread Spectrum Signals

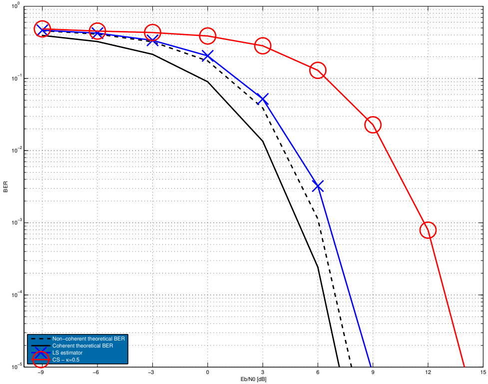

Figure: Bit-error-rate – downsampling at the receiver.

RF Communication

Compressed Sensing-Based Direct Conversion Receiver

- Computational power of modern data receivers enables moving more processing from the analog to the digital domain.

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- The filtered, down-converted radio signal is randomly sampled with a sub-Nyquist average sampling frequency.

RF Communication

Compressed Sensing-Based Direct Conversion Receiver

- Computational power of modern data receivers enables moving more processing from the analog to the digital domain.
- Use compressed sensing to relax the analog filtering requirements in a direct conversion receiver.
- The filtered, down-converted radio signal is randomly sampled with a sub-Nyquist average sampling frequency.
- Exploits frequency-domain sparsity of the down-converted radio signals.

RF Communication

Compressed Sensing-Based Direct Conversion Receiver

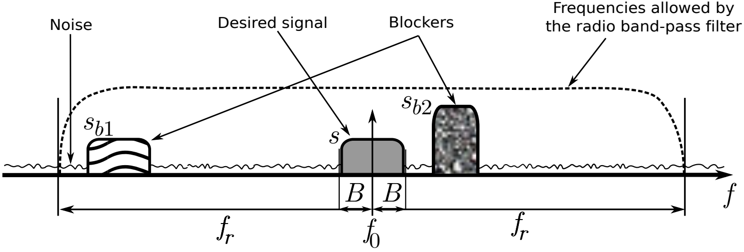

Figure: Spectral content around desired signal.

RF Communication

Compressed Sensing-Based Direct Conversion Receiver

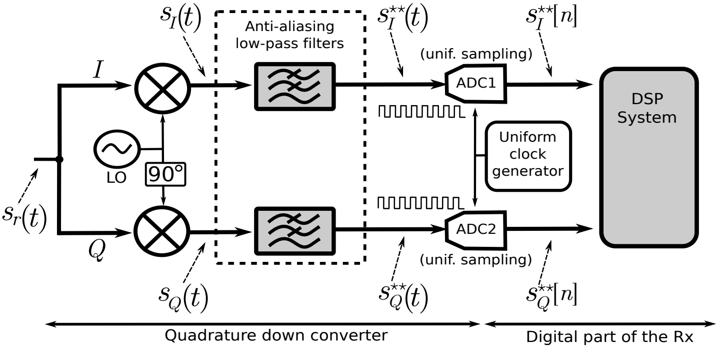

Figure: Ordinary direct conversion receiver.

RF Communication

Compressed Sensing-Based Direct Conversion Receiver

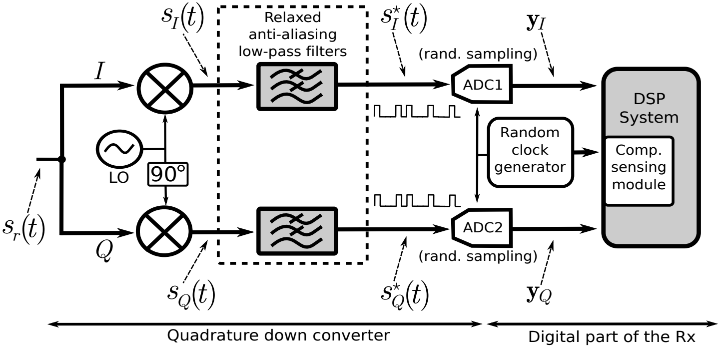

Figure: Proposed CS-based direct conversion receiver.

Agenda

Projects and People

RF Communication

Reconstruction

D/A Conversion

Reconstruction

Correlation Between Measurements and Noise

- What happens when compressed measurements are affected by noise correlated with the measurements?

$$\bar{\mathbf{y}} = \mathbf{A}\mathbf{x}$$
$$\mathbf{y} = \bar{\mathbf{y}} + \mathbf{n},$$

Reconstruction

Correlation Between Measurements and Noise

- What happens when compressed measurements are affected by noise correlated with the measurements?

$$\bar{\mathbf{y}} = \mathbf{A}\mathbf{x}$$
$$\mathbf{y} = \bar{\mathbf{y}} + \mathbf{n},$$

- Where does this happen?
  For example when quantizing measurements at low resolution.

Reconstruction

Correlation Between Measurements and Noise

- Linear correlated noise model:

$$\mathbf{y} = \alpha \mathbf{A} \mathbf{x} + \mathbf{w},$$

Reconstruction

Correlation Between Measurements and Noise

- Linear correlated noise model:

$$\mathbf{y} = \alpha \mathbf{A}\mathbf{x} + \mathbf{w},$$

- Leads to noise correlated with the measurements

$$\begin{aligned}\mathbf{n} &= \mathbf{y} - \bar{\mathbf{y}} \\ &= (\alpha - 1)\bar{\mathbf{y}} + \mathbf{w}\end{aligned}$$

Reconstruction

Correlation Between Measurements and Noise

- What to do about it?

Reconstruction

Correlation Between Measurements and Noise

- What to do about it?
- We can simply rescale the estimate of **x** after BPDN reconstruction:

$$\hat{\mathbf{x}} = \frac{1}{\alpha} \underset{\mathbf{v}:\ \|\mathbf{y}-\mathbf{A}\mathbf{v}\|_2 \leq \epsilon}{\operatorname{argmin}} \|\mathbf{v}\|_1. \qquad (1)$$

Reconstruction

Correlation Between Measurements and Noise

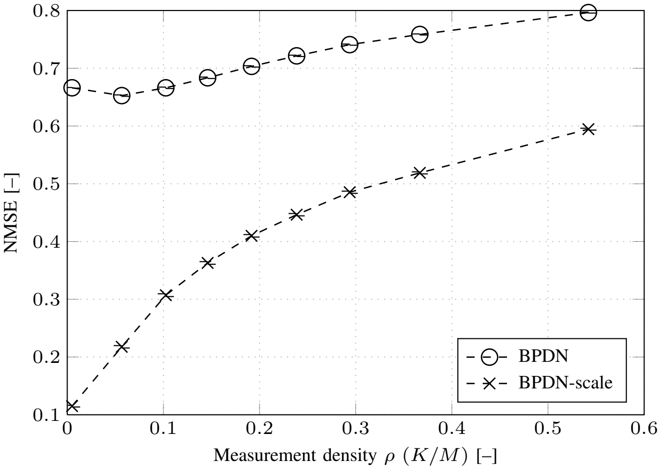

Figure: Example of improvement for 1 bit/sample quantization.

Agenda

Projects and People

RF Communication

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D/A Conversion

D/A Conversion

Compressive Parameter Estimation

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- We are currently investigating such models for use in time delay estimation and sinosoidal parameter estimation.

D/A Conversion

Compressive Parameter Estimation

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- Especially interesting: manifold models.
- Allow parameters drawn from a continuous space, rather than from a discrete dictionary as usual.
- We are currently investigating such models for use in time delay estimation and sinosoidal parameter estimation.
- We show that it is possible to use compressive sensing and still attain good mean squared error on the parameter estimate.

D/A Conversion

Quantization in CS with a Fixed Bit Budget

- Investigation of reconstruction performance in compressive sensing with quantized measurements.

D/A Conversion

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- Trade-off between quantizer resolution and number of compressed measurements for fixed total numbers of bits.

D/A Conversion

Quantization in CS with a Fixed Bit Budget

- Investigation of reconstruction performance in compressive sensing with quantized measurements.
- Trade-off between quantizer resolution and number of compressed measurements for fixed total numbers of bits.
- We compare two methods by Laska et al. tailored to saturated measurements for the Basis Pursuit De-Noising method.

D/A Conversion

Quantization in CS with a Fixed Bit Budget

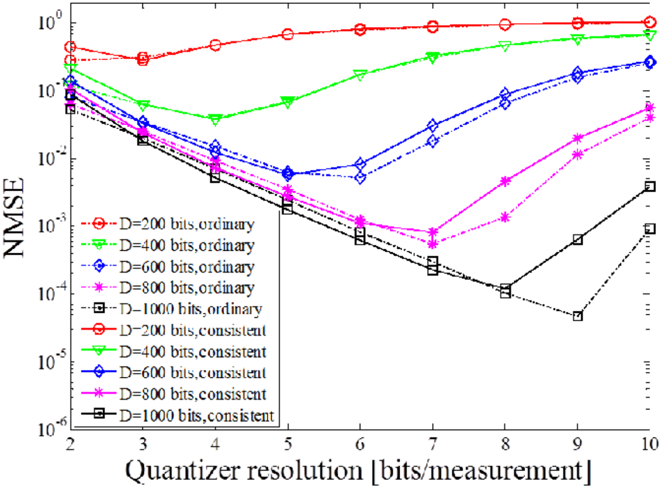

Figure: Normalized mean-squared error. Saturation rate 5%.

D/A Conversion

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- We propose reserving quantization indices to mark saturated measurements.
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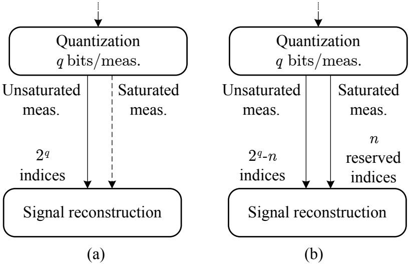

Figure: Reserving quantizer indices for indicating saturation.

D/A Conversion

Quantization in CS with a Fixed Bit Budget

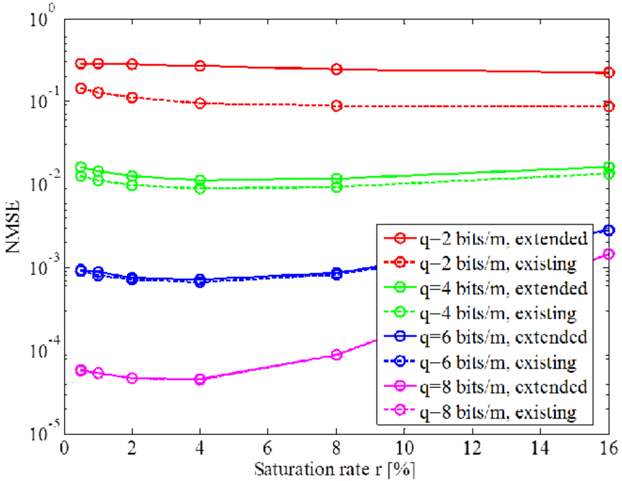

Figure: Normalized mean-squared error vs. quantizer resolution.

D/A Conversion

Filter Imperfections in Random Demodulator

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D/A Conversion

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Filter Imperfections in Random Demodulator

- Applying compressed sensing (CS) to continuous signals: analog sampling front-end providing a signal representation compatible with CS.
- The random demodulator provides pseudo-random linear projections of the analog input signal.
- The analog front-end has to be modeled accurately in the reconstruction algorithm.

D/A Conversion

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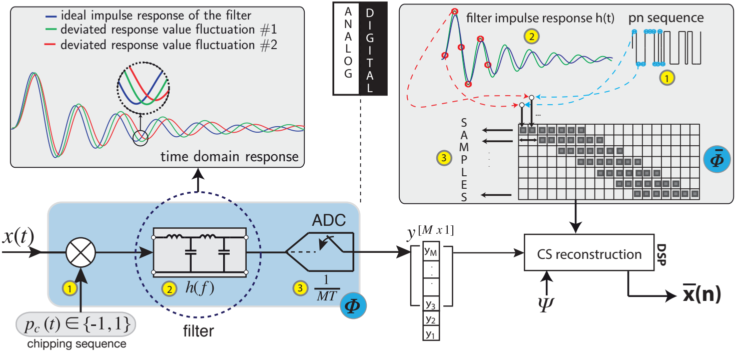

D/A Conversion

Measurement Matrix Deviations due to Filter Imperfections in Random Demodulator

- Simulations of imperfect filter: component variation 5% and 10% for capacitors and inductors respectively.
- Simulations of 16 worst-case scenarios using 4th order Butterworth filter indicated up to 40 dB loss in SNR

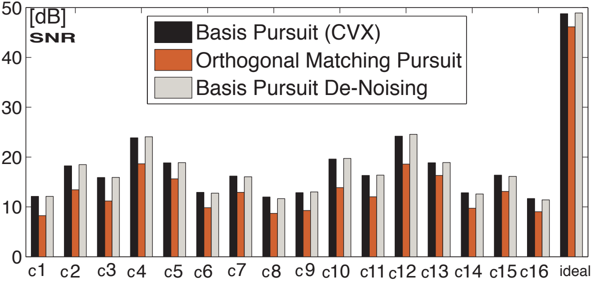

Figure: Reconstruction error corner cases for imperfect filter.

D/A Conversion

Calibration of Filter Imperfections in the Random Demodulator

- Mismatch considered an additive error in the discretized impulse response: $\mathbf{y} = (\mathbf{A} + \mathbf{E})\mathbf{x}$

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- Estimate error by sampling a known signal, enabling least-squares estimation of the impulse response error.
- Error and known problem structure are used to correct the measurement matrix.
- Simulation results demonstrate the effectiveness of the calibration technique even for highly deviating low-pass filter responses.

D/A Conversion
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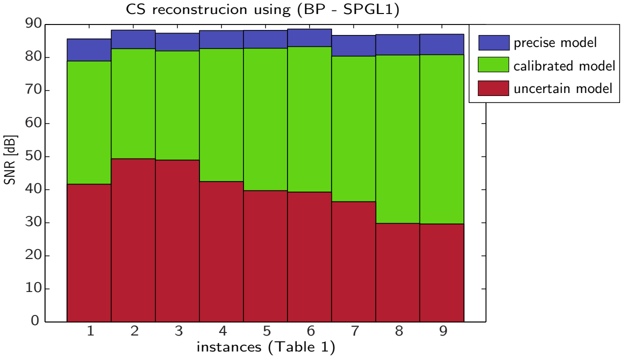

Figure: Example: improvements from calibration of all filter components deviating up to 2%.