Blind deconvolution of PET images using anatomical priors

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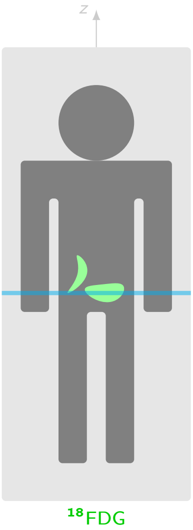
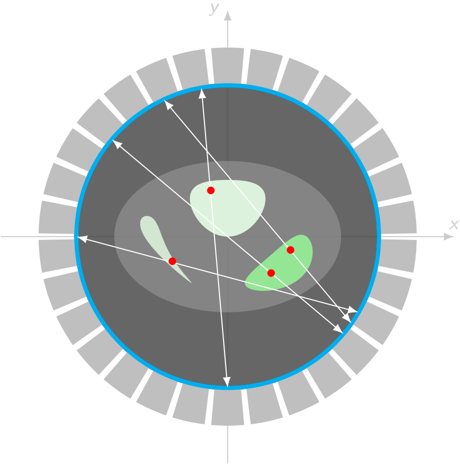
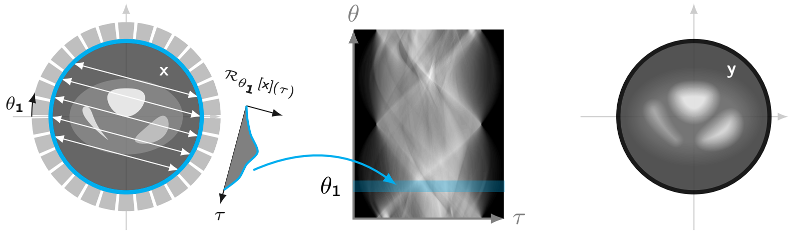
A radioactive analogue of glucose is injected in the patient





PET images provide access to the metabolic activity of a patient



Tomographic acquisition — $\mathbf{x} \mapsto \mathcal{R}_\theta[\mathbf{x}](\tau)$

Degradation — $\mathcal{D}(\mathcal{R}\mathbf{x})$

Observed image $\mathbf{y}$ — $\mathcal{R}^{-1}(\mathcal{D}(\mathcal{R}\mathbf{x}))$

Context. No access to raw data: processing in a post-reconstruction phase.

Goal. Quality improvement of PET images for a better delineation of the tumor volumes.

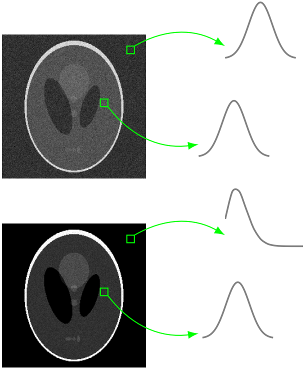
Photon counting introduces multiplicative noise

Gaussian noise
due to the electronics

additive

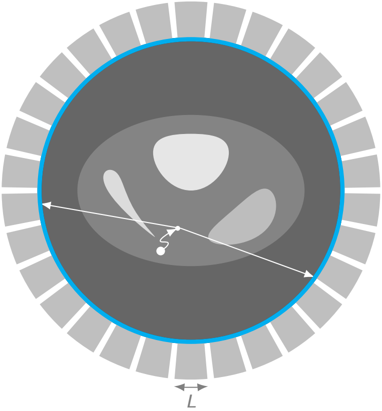
Poisson noise
due to the counting process

multiplicative



PET images are potentially affected by both types of noise

External and internal factors degrade the image resolution



Free travel of positron before annihilation

Imperfect anticollinearity

Size and width of the detectors

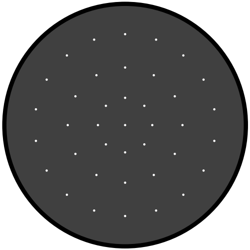
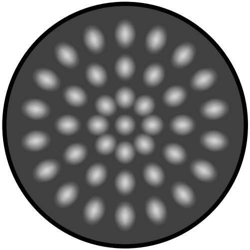
Incident angle and depth of interaction of the photon i

...

Type of tissue (attenuation)

Patient movement

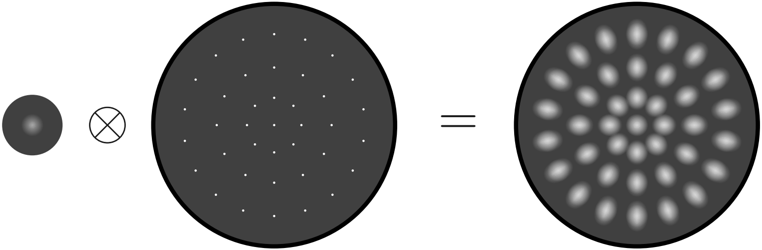
The *blurring* is assumed linear and uniform in the FoV

$f_{blur}($  $) =$ 

Function f_{blur} is very general

The resolution may change for different locations in the FoV

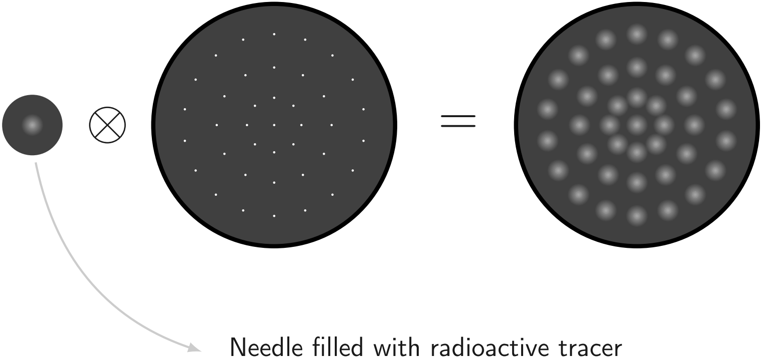
The *blurring* is assumed linear and uniform in the FoV



Hypotheses : **linear** function
same effect for all pixels

f_{blur} is written as a convolution with a kernel **h**

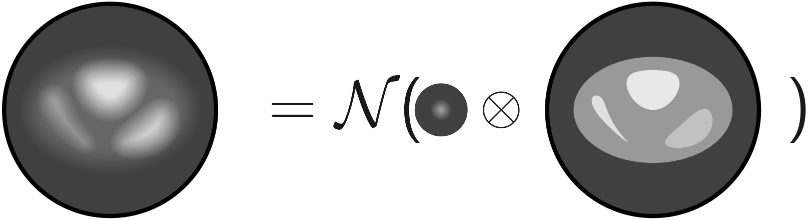
The kernel **h** is estimated by imaging a linear source



Needle filled with radioactive tracer
Acquisition at the center of the FoV

Approximation by an isotropic Gaussian function

The forward model links the observation to the original image



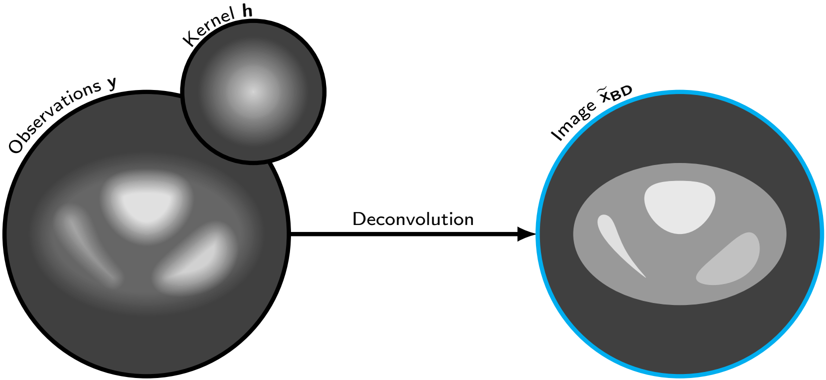
Forward Model

Hypothesis. **h** uniform in the FoV

$$\mathbf{y} = \mathcal{N}(\mathbf{h} \otimes \mathbf{x}),$$

where $\mathcal{N}(\cdot)$ is a noise operator

The inverse problem aims at estimating $\mathbf{x}$ from $\mathbf{y}$



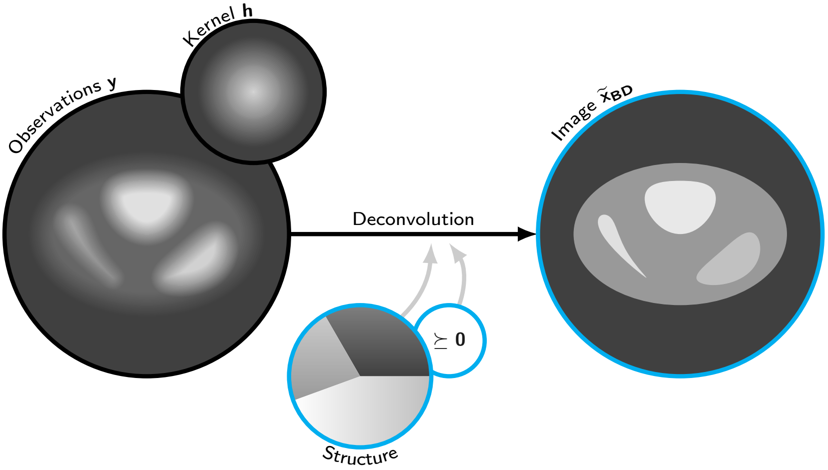
Depends on the noise statistics

$\underset{\mathbf{x}}{\text{minimize}}\ \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y})$ **ill-posed!**

Related to the image

e.g., Gaussian noise $\Rightarrow$ $\text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) = \frac{1}{2}\|\mathbf{h} \otimes \mathbf{x} - \mathbf{y}\|_2^2$

We regularize the problem by adding prior information on the image



Related to the image

$$\underset{\mathbf{x}}{\text{minimize}}\ \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + S(\mathbf{x}), \quad \text{subject to} \quad \mathbf{x} \succeq \mathbf{0}$$

$S(\mathbf{x}) \Rightarrow$ TV norm, TGV norm, $\|\boldsymbol{\Psi}^* \cdot\|_1$

Solving the deconvolution convex problem

Regularized deconvolution problem

Hypotheses. $\mathbf{h}$ uniform in the FoV

$$\underset{\mathbf{x}}{\text{minimize}} \quad \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + S(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \succeq \mathbf{0}$$

$$\underset{\mathbf{x}}{\text{minimize}} \sum_{j=1}^{L} F_j(\mathbf{K}_j \mathbf{x}) + G(\mathbf{x}) \qquad F_j, G : \mathbb{R}^N \to (-\infty, \infty]$$
$$\mathbf{K}_j : \mathbb{R}^N \to \mathbb{R}^{W_j}$$

Generalized Forward-Backward (GFB)

- GFB: G differentiable & $\mathbf{K}_j$ tight frame or $(\mathbf{K}_j^* \mathbf{K}_j + \mathbf{I})$ easily invertible

G: convex and differentiable
F_j: convex, proper, l.s.c.
$H_j(\cdot) := F_j(\mathbf{K}_j \ \cdot)$

$$\begin{cases} \mathbf{s}_j^{(k+1)} &= \mathbf{s}_j^{(k)} + \lambda(\text{prox}_{\frac{\gamma}{w_j} H_j}(2\mathbf{x}^{(k)} - \mathbf{s}_j^{(k)} - \gamma \nabla G(\mathbf{x}^{(k)})) - \mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} &= \sum_{j=1}^{L} w_j \mathbf{s}^{(k+1)} \end{cases}$$

$$\text{prox}_{H_j}(\mathbf{z}) \quad \Rightarrow \quad \begin{cases} \mathbf{K}_j \text{ tight frame} \Rightarrow \mathbf{K}_j \mathbf{K}_j^* = \nu \mathbf{I} \\ (\mathbf{K}_j^* \mathbf{K}_j + \mathbf{I}) \text{ easily invertible} \end{cases}$$

[Raguet et al. 2013]

Solving the deconvolution convex problem

Regularized deconvolution problem

Hypotheses. **h** uniform in the FoV

$$\underset{\mathbf{x}}{\text{minimize}} \quad \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + S(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \succeq \mathbf{0}$$

$$\underset{\mathbf{x}}{\text{minimize}} \sum_{j=1}^{L} F_j(\mathbf{K}_j\mathbf{x}) + G(\mathbf{x}) \qquad F_j, G : \mathbb{R}^N \to (-\infty, \infty]$$
$$\mathbf{K}_j : \mathbb{R}^N \to \mathbb{R}^{W_j}$$

Alternating Direction Method of Multipliers (ADMM)

F_j: convex, proper, l.s.c.

$G = 0$

- GFB: G differentiable & $\mathbf{K}_j$ tight frame or $(\mathbf{K}_j^*\mathbf{K}_j + \mathbf{I})$ easily invertible
- ADMM: $(\sum_{j=1}^{L} \mathbf{K}_j^*\mathbf{K}_j)$ easily invertible

$$\begin{cases} \mathbf{x}^{(k+1)} = (\sum_{j=1}^{L} \mu_j \mathbf{K}_j^*\mathbf{K}_j)^{-1} \sum_{j=1}^{L} \mu_j \mathbf{K}_j^* (\mathbf{u}_j^{(k)} + \mathbf{d}_j^{(k)}) \\ \mathbf{u}_j^{(k+1)} = \text{prox}_{F_j/\mu_j}(\mathbf{K}_j\mathbf{x}^{(k+1)} - \mathbf{d}_j^{(k)}) \\ \mathbf{d}_j^{(k+1)} = \mathbf{d}_j^{(k)} - (\mathbf{K}_j\mathbf{x}^{(k+1)} - \mathbf{u}_j^{(k)}) \end{cases}$$

[Almeida & Figueiredo 2013]

Solving the deconvolution convex problem

Regularized deconvolution problem

Hypotheses. $\mathbf{h}$ uniform in the FoV

$$\underset{\mathbf{x}}{\text{minimize}} \quad \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + S(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \succeq \mathbf{0}$$

$$\underset{\mathbf{x}}{\text{minimize}} \sum_{j=1}^{L} F_j(\mathbf{K}_j \mathbf{x}) + G(\mathbf{x}) \qquad F_j, G : \mathbb{R}^N \to (-\infty, \infty]$$
$$\mathbf{K}_j : \mathbb{R}^N \to \mathbb{R}^{W_j}$$

Chambolle-Pock (CP) Algorithm

F_j, G: convex, proper, l.s.c.

- GFB: G differentiable & $\mathbf{K}_j$ tight frame or $(\mathbf{K}_j^* \mathbf{K}_j + \mathbf{I})$ easily invertible
- ADMM: $(\sum_{j=1}^{L} \mathbf{K}_j^* \mathbf{K}_j)$ easily invertible
- CP: general but slow convergence

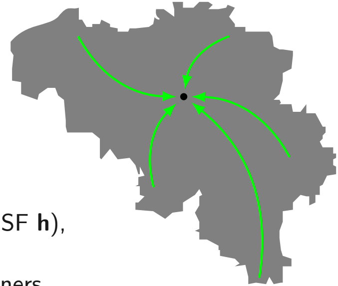
$$\begin{cases} \mathbf{s}_j^{(k+1)} = \text{prox}_{\nu F_j^*}\left(\mathbf{s}_j^{(k)} + \nu \mathbf{K}_j \overline{\mathbf{x}}^{(k)}\right), \; j \in \{1, \cdots L\} \\ \mathbf{x}^{(k+1)} = \text{prox}_{\frac{\mu}{L} G}\left(\mathbf{x}^{(k)} - \frac{\mu}{L} \sum_{j=1}^{L} \mathbf{K}_j^* \mathbf{s}_j^{(k+1)}\right) \\ \overline{\mathbf{x}}^{(k+1)} = 2\,\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)} \end{cases}$$

[Chambolle & Pock 2011]

An *a priori* estimation of **h** is not always possible

- **h** does not follow a specific parametric model
- **h** is not necessarily isotropic
- the actual medium is different to the one used in the estimation of **h** (related to the patient)
- it is not always possible to access the scanner to estimate **h**

A multicentric study treats very diverse data



Context

Reconstructed images from different clinical centres

Challenge

No access to scanners properties (*e.g.*, to the PSF **h**),
to raw data

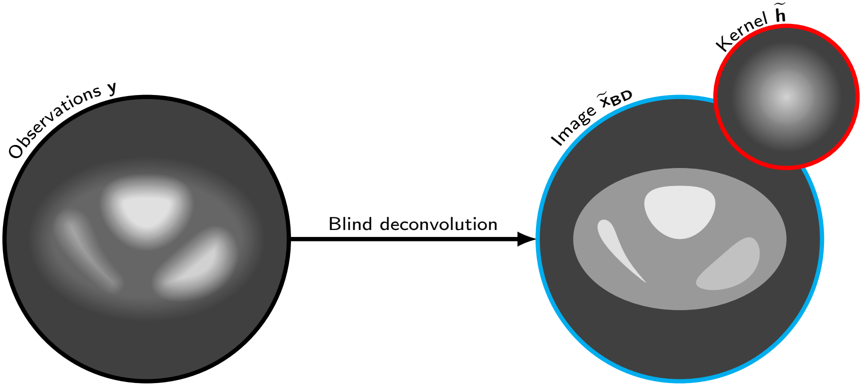
Access to CT images from combined PET/CT scanners

Research question

How to restore an image and retrieve the PSF of the acquisition device from corrupted observations and anatomical images from another imaging modality?

What to do if it is impossible to access kernel $\mathbf{h}$?

The objective is to simultaneously estimate **h** and restore the image

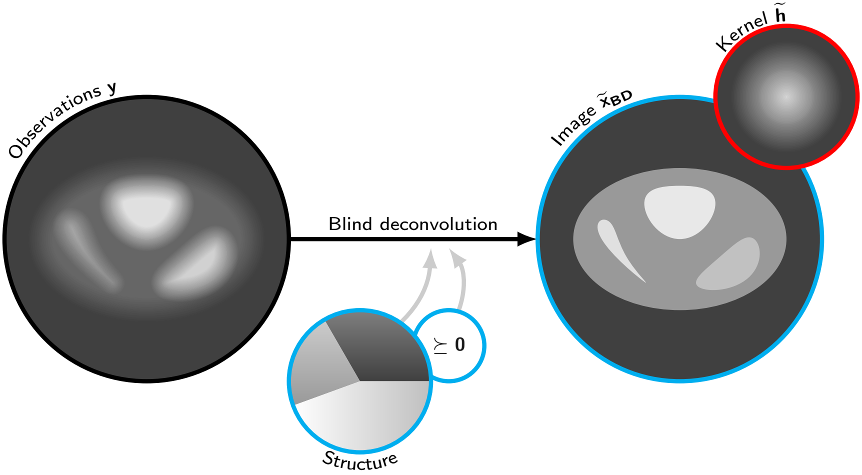


$$\underset{\mathbf{x},\mathbf{h}}{\text{minimize}}\ \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y})$$

ill-posed!

- Related to the image
- Related to the kernel

We already have some prior information on the image



$$\underset{\mathbf{x},\mathbf{h}}{\text{minimize}}\ \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + S(\mathbf{x}), \quad \text{subject to} \quad \mathbf{x} \succeq \mathbf{0}$$

Related to the image

Related to the kernel

We impose some weak constraints on the kernel $\mathbf{h}$

Non-negativity : $\mathbf{h} \succeq \mathbf{0}$

Flux preservation : $\sum_{i=1}^{N} |h_i| = 1$

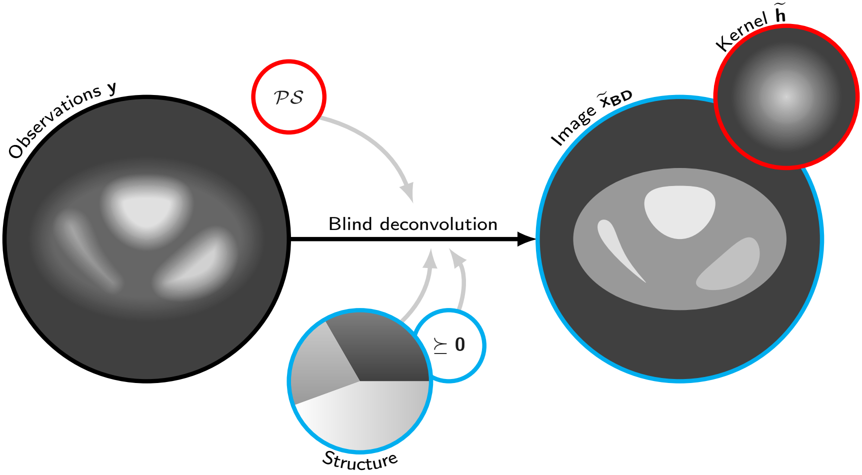
$20 \otimes \mathbf{h} =$

		1		
	1	2	1	
1	2	4	2	1
	1	2	1	
		1		

or *photometric invariance property*...

} Probability Simplex $\mathcal{PS}$

We impose some weak constraints on the kernel $\mathbf{h}$

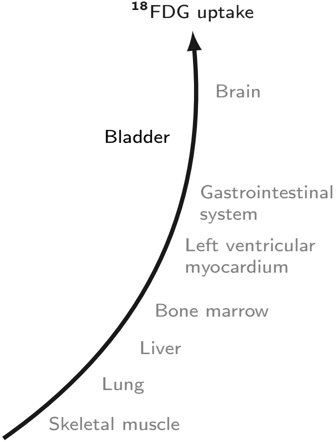


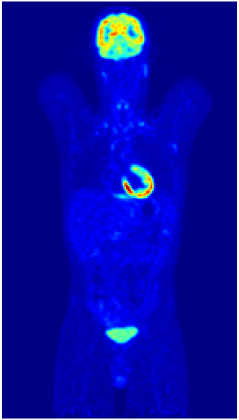
$$\underset{\mathbf{x},\mathbf{h}}{\text{minimize}}\ \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + \mathcal{S}(\mathbf{x}), \quad \text{subject to} \quad \mathbf{x} \succeq \mathbf{0}, \quad \mathbf{h} \in \mathcal{PS}$$

Related to the image

Related to the kernel

The usual distribution of ^{18}FDG in the body is known

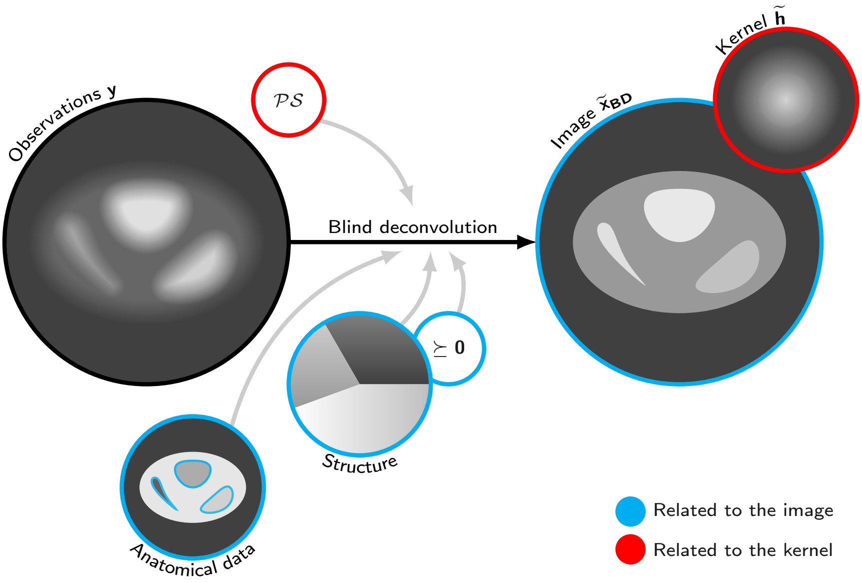




PET-scan

CT-scan

Including CT data in the blind deconvolution problem



The blind deconvolution problem is non convex

Regularized blind deconvolution problem

Hypotheses. $\mathbf{h}$ uniform in the FoV

$$\underset{\mathbf{x},\mathbf{h}}{\text{minimize}} \quad \text{dist}(\mathbf{h} \otimes \mathbf{x}, \mathbf{y}) + S(\mathbf{x})$$

subject to $\mathbf{x}$ constant in ,  & 

$$\mathbf{x} \succeq \mathbf{0}, \quad \mathbf{h} \in \mathcal{PS}$$

Solved through an alternated minimization

$$(\mathbf{x}^{(k)}, \mathbf{h}^{(k)}) \to (\mathbf{x}^{(k+1)}, \mathbf{h}^{(k)}) \to (\mathbf{x}^{(k+1)}, \mathbf{h}^{(k+1)})$$

Solving the blind deconvolution non-convex problem

Proximal alternating algorithm

$$\begin{cases} \mathbf{x}^{(k+1)} & = \operatorname{argmin}_{\mathbf{x}} \; L(\mathbf{x},\; \mathbf{h}^{(k)}) + \frac{\lambda_x^{(k)}}{2}\|\mathbf{x} - \mathbf{x}^{(k)}\|_2^2 \\ \mathbf{h}^{(k+1)} & = \operatorname{argmin}_{\mathbf{h}} \; L(\mathbf{x}^{(k+1)}, \mathbf{h}) + \frac{\lambda_h^{(k)}}{2}\|\mathbf{h} - \mathbf{h}^{(k)}\|_2^2 \end{cases}$$

- $L(\mathbf{x}, \mathbf{h})$: objective function including cost function and constraints
- λ_x, λ_h: cost-to-move parameters

Conditions

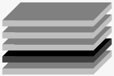
- Structured problem: $L(\mathbf{x}, \mathbf{h}) := F(\mathbf{x}) + Q(\mathbf{x}, \mathbf{h}) + G(\mathbf{h})$
  - $F : \mathbb{R}^N \to (-\infty, +\infty]$ & $G : \mathbb{R}^W \to \mathbb{R} \cup (-\infty, +\infty]$ proper l.s.c.
  - $Q : \mathbb{R}^N \times \mathbb{R}^W \to \mathbb{R}$ smooth C^1 function with ∇Q Lipschitz continuous on bounded subsets of $\mathbb{R}^N \times \mathbb{R}^W$
- $L(\mathbf{x}, \mathbf{h})$ bounded below, $L(\cdot, \mathbf{h})$ proper & λ_x, λ_h bounded
- $L(\mathbf{x}, \mathbf{h})$ satisfies the Kurdyka-Lojasiewicz property $\Rightarrow$ e.g., semi-algebraic function
- $(\mathbf{x}^{(k+1)}, \mathbf{h}^{(k+1)})$ bounded

$\Rightarrow (\mathbf{x}^{(k+1)}, \mathbf{h}^{(k+1)})$ converges to a critical point of $L(\mathbf{x}, \mathbf{h})$

[Attouch et al. 2010, Attouch et al. 2013, González et al. 2016]

The problems in hand

Step 1: Blind deconvolution problem



$$\begin{aligned} \underset{\mathbf{x},\mathbf{h}}{\text{minimize}} \quad & \frac{\rho}{2}\|\mathbf{h}\otimes\mathbf{x}-\mathbf{y}\|_2^2 + \mathrm{TV}(\mathbf{x}) \\ \text{subject to} \quad & (\nabla\mathbf{x})_i = 0 \quad \text{if } i \in \Omega_1, \Omega_2, \ldots, \\ & \mathbf{x} \succeq \mathbf{0}, \quad \mathbf{h} \in \mathcal{PS}. \end{aligned}$$

Objective function: $L_{\mathrm{BD}}(\mathbf{x}, \mathbf{h}, \rho)$

$\widetilde{\mathbf{h}}_{\mathbf{BD}}$ $\widetilde{\mathbf{x}}_{\mathbf{BD}}$

Step 2: Non blind deconvolution problem

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \frac{\rho}{2}\|\mathbf{h}\otimes\mathbf{x}-\mathbf{y}\|_2^2 + \mathrm{TV}(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{x} \succeq \mathbf{0}. \end{aligned}$$

Objective function: $L_{\mathrm{NBD}}(\mathbf{x}, \mathbf{h}, \rho)$

The problems in hand

Step 1: Blind deconvolution problem

Set: $\mathbf{x}^{(1)} = \mathbf{y}$; $\mathbf{h}^{(1)} = \delta_0$; $\rho^{(1)} = \sigma\sqrt{2\log N}$; $\varepsilon = \sigma\sqrt{N+2\sqrt{N}}$; $\text{It} = 5$; $\text{It2} = 10^4$

for $t = 1$ to It **do**

1: Proximal Alternating Algorithm with $\mathbf{x}^{(0)} = \mathbf{x}^{(t)}$, $\mathbf{h}^{(0)} = \mathbf{h}^{(t)}$ and $\rho^{(0)} = \rho^{(t)}$

for $k = 1$ to It2 **do**

$$\mathbf{x}^{(k+1)} = \text{argmin}_{\mathbf{x}}\ L_{\text{BD}}(\mathbf{x},\ \mathbf{h}^{(k)}, \rho^{(k)}) + \frac{\lambda_x^{(k)}}{2}\|\mathbf{x} - \mathbf{x}^{(k)}\|_2^2 \quad \Rightarrow \textbf{CP}$$

$$\mathbf{h}^{(k+1)} = \text{argmin}_{\mathbf{h}}\ L_{\text{BD}}(\mathbf{x}^{(k+1)}, \mathbf{h}) + \frac{\lambda_h^{(k)}}{2}\|\mathbf{h} - \mathbf{h}^{(k)}\|_2^2 \quad \Rightarrow \textbf{GFB}$$

Return: $\mathbf{x}^{(t+1)} = \mathbf{x}^{(k+1)}$ and $\mathbf{h}^{(t+1)} = \mathbf{h}^{(k+1)}$

2: Parameter update: $\mathbf{r}^{(t+1)} = \mathbf{y} - \mathbf{h}^{(t+1)} \otimes \mathbf{x}^{(t+1)} \Rightarrow \rho^{(t+1)} = \rho^{(t)}(\varepsilon/\|\mathbf{r}^{(t+1)}\|_2)$

Return: $\widetilde{\mathbf{x}}_{\text{BD}} = \mathbf{x}^{(t+1)}$ and $\widetilde{\mathbf{h}}_{\text{BD}} = \mathbf{h}^{(t+1)}$

Step 2: Non blind deconvolution problem

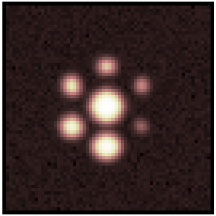
Set: $\mathbf{x}^{(1)} = \mathbf{y}$; $\rho = \sigma\sqrt{2\log N}$; $\text{It} = 10^4$

for $k = 1$ to It **do** $\quad \mathbf{x}^{(k+1)} = \text{argmin}_{\mathbf{x}}\ L_{\text{NBD}}(\mathbf{x}, \widetilde{\mathbf{h}}_{\text{BD}}) \quad \Rightarrow$ **CP, GFB or ADMM**

Return: $\widetilde{\mathbf{x}}_{\text{NBD}} = \mathbf{x}^{(k+1)}$

What does this mean in practice?

Some synthetic results to validate the method



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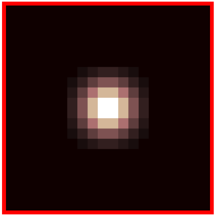
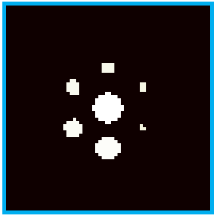


$\otimes$



$+ \; \mathbf{n}$

Observation $\mathbf{y}$

Kernel $\mathbf{h}$

Simulates the kernel used in practice: isotropic Gaussian

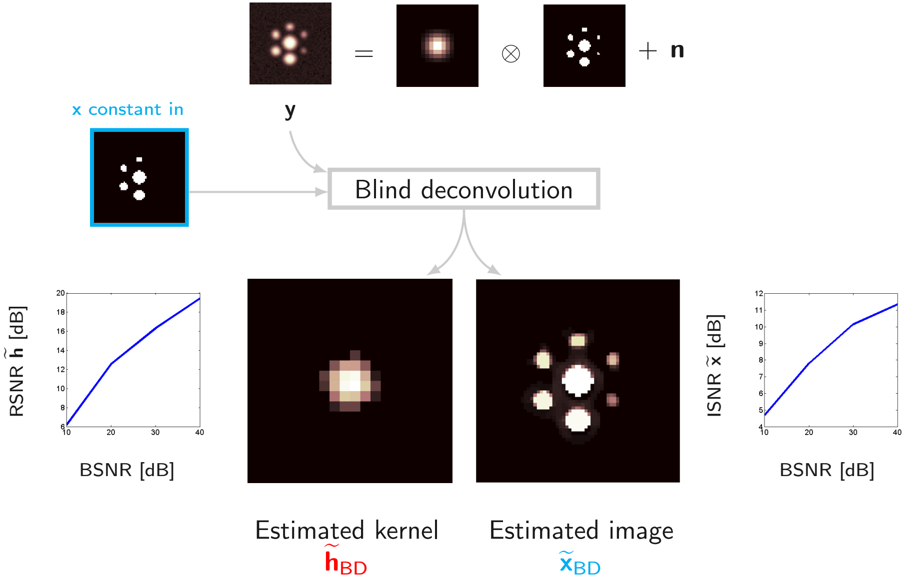
Original image $\mathbf{x}$

Simulates a phantom on cylinders filled with ^{18}FDG

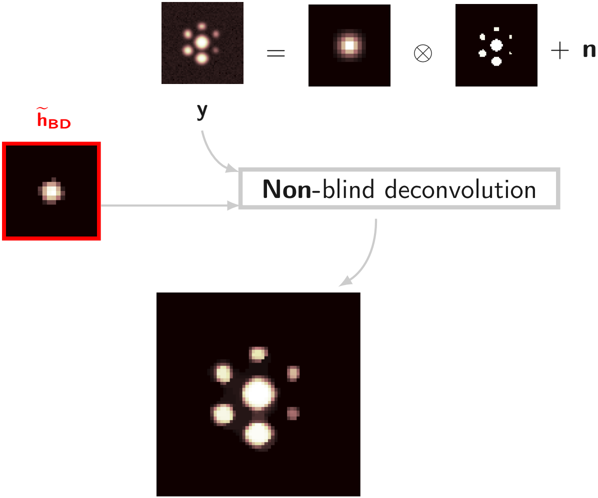
Noise

additive white Gaussian

Blind deconvolution to estimate the image and the kernel



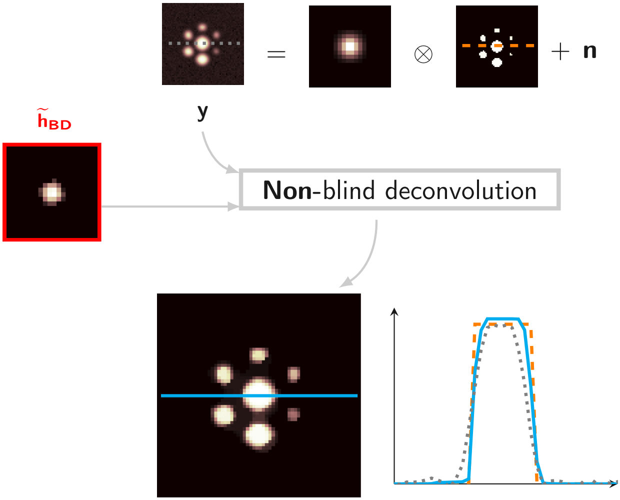
Validation of the estimated kernel



Estimated image

$\widetilde{x}_{NBD}$

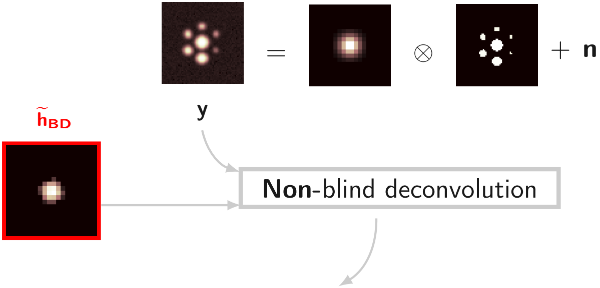
Validation of the estimated kernel

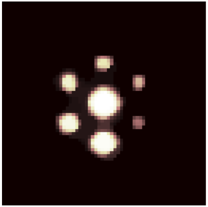


Estimated image $\widetilde{\mathbf{x}}_{NBD}$

Profiles 1-D of $\mathbf{x}$, $\widetilde{\mathbf{x}}_{NBD}$ and $\mathbf{y}$

Validation of the estimated kernel





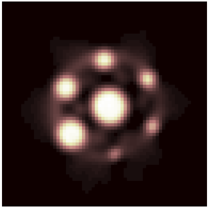
	ISNR [dB]	# iter	time [s]
GFB	3.99	1162	6.07
ADMM	**4.53**	**960**	**2.57**
CP	4.08	6240	10.86

Estimated image
$\widetilde{\mathbf{x}}_{\mathrm{NBD}}$

Comparison of methods

Some experimental data - PET/CT Cliniques universitaires Saint-Luc

Observation of a phantom with cylinders filled with ^{18}FDG

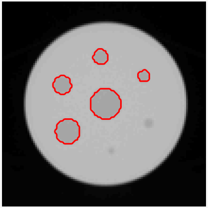


$= \; ? \; \otimes \; ? \; + \mathbf{n}$

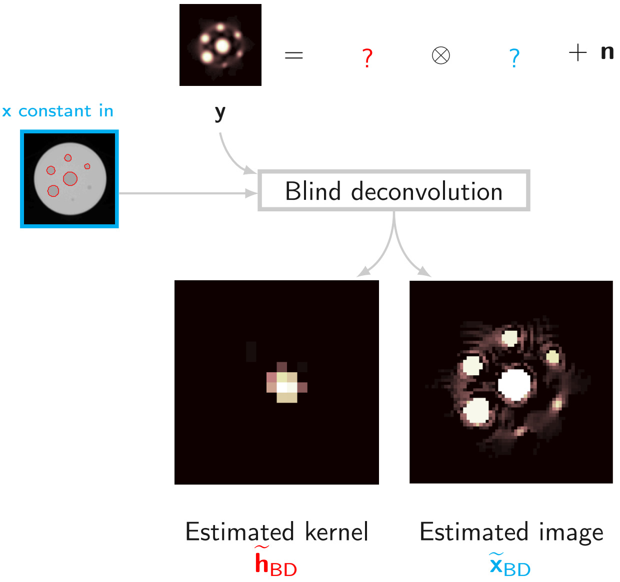
Observation $\mathbf{y}$ | Kernel $\mathbf{h}$ | Original image $\mathbf{x}$ | AWGN

Prior information

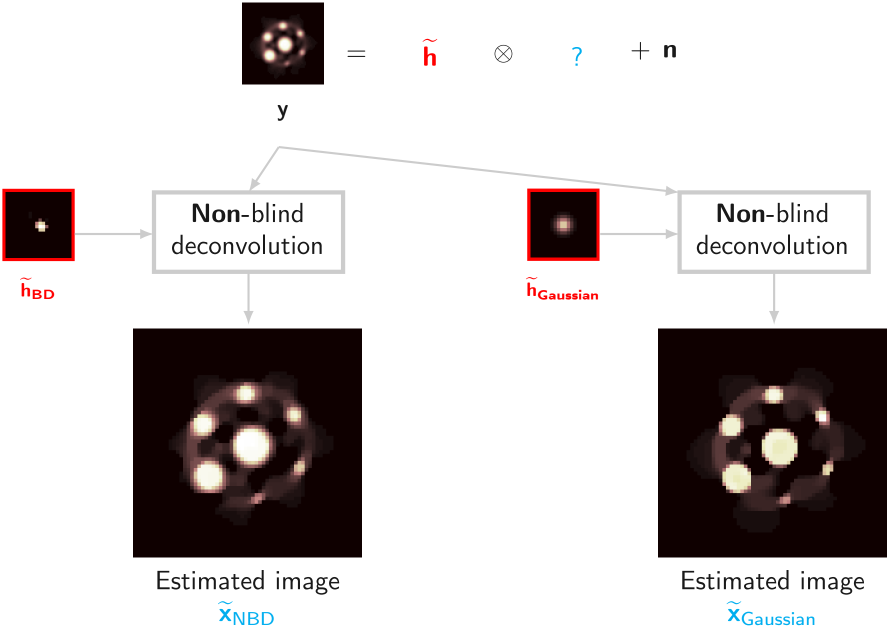
CT image to find a mask for the areas with constant intensity in the PET image



Blind deconvolution to estimate the image and the kernel



Validation of the estimated kernel



Perspectives

- Consider Poisson noise and/or mixed Poisson-Gaussian
- Consider the 3-D volume instead of the 2-D reconstruction per slice
- Add constraints to the structure of the kernel
- Work with patient data

Blind deconvolution of PET images using anatomical priors

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