A DECONVOLUTION PROBLEM IN ASTRONOMY

Image and Signal Processing Seminars

Adriana González

Joint work with Prof. Laurent Jacques and in collaboration with Dr. Véronique Delouille from the Royal Observatory of Belgium.

UCL/ICTEAM/ELEN

March 27, 2013

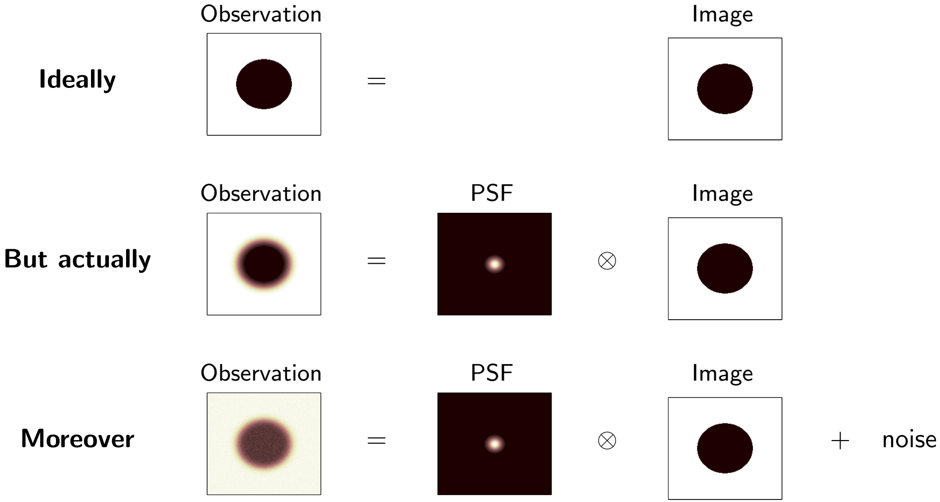
Outline

1. Introduction
2. Problem definition
3. Data preprocessing
4. Reconstruction
5. Noise estimation
6. Results
7. Future works

Outline

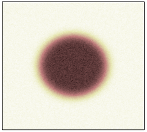
1. Introduction
2. Problem definition
3. Data preprocessing
4. Reconstruction
5. Noise estimation
6. Results
7. Future works

Imaging distortions



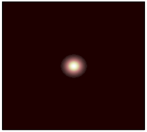
Blind deconvolution problem

Observation



$=$

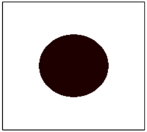
PSF



?

$\otimes$

Image



?

$+ \quad n$

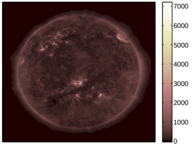
$\Downarrow$

What we need

Outline

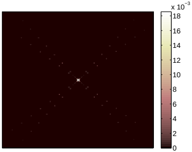
1. Introduction
2. Problem definition
3. Data preprocessing
4. Reconstruction
5. Noise estimation
6. Results
7. Future works

Observation



$=$

PSF



$\otimes$

Image

?

$\otimes \quad + \quad n$

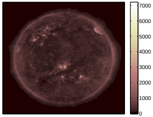
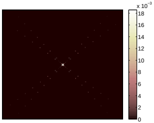
Easy to solve!!! But...
Observations are also affected by stray light.

Problem definition

- Stray light

  ⇒ undesired light in the optical system.

  ⇒ modeled as an additional filter convolving the image.

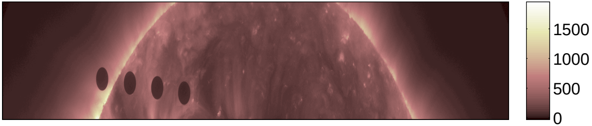
Observation		PSF		Stray Light		Image		
	$=$		$\otimes$	**?**	$\otimes$	**?**	$\otimes$	$+\ n$

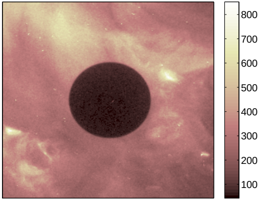
Problem

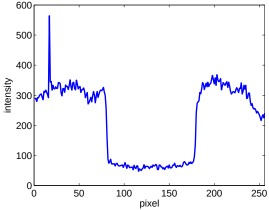
Stray light correction to study low intensity regions in the Sun.

Available information

- Venus transit: June 6, 2012.







Prior information

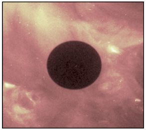
Venus is a **uniformly black disk** with a known diameter.

Redefining the problem

1. Stray light estimation: Estimating f_2

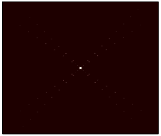
$$y = f_1 \otimes \hat{g} \otimes f_2 + n_1 \quad \Rightarrow \quad \mathbf{y} = \mathbf{F}_1 \hat{\mathbf{G}}\, \mathbf{f}_2 + \mathbf{n}_1$$

y



$=$

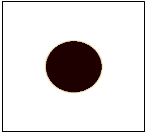
f_1



PSF

$\otimes$

$\hat{g}$



$\otimes$

f_2

?

Stray Light

$+ \quad n_1$

2. Stray light correction: Estimating g

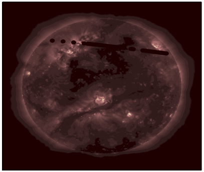
$$y = f_1 \otimes f_2 \otimes g + n_2 \quad \Rightarrow \quad \mathbf{y} = \mathbf{F}_1 \mathbf{F}_2\, \mathbf{g} + \mathbf{n}_2$$

$$\mathbf{y} = \mathbf{F} \;\; \mathbf{g} + \mathbf{n}_2$$

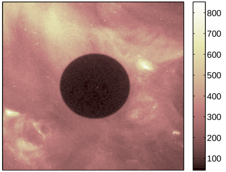
Outline

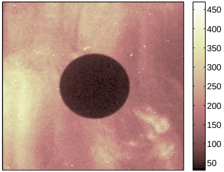
1. Introduction
2. Problem definition
3. Data preprocessing
4. Reconstruction
5. Noise estimation
6. Results
7. Future works

$$\mathbf{I}_V = \mathbf{I}_{V_0} \otimes \mathbf{f}, \qquad \mathbf{f} = \mathbf{f}_1 \otimes \mathbf{f}_2$$



$$\mathbf{V}_j = \mathbf{S}_j \mathbf{I}_V = \mathbf{S}_j(\mathbf{I}_{V_0} \otimes \mathbf{f}) = (\mathbf{S}_j \mathbf{I}_{V_0}) \otimes \mathbf{f}$$

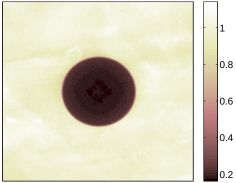




Optimized weighting

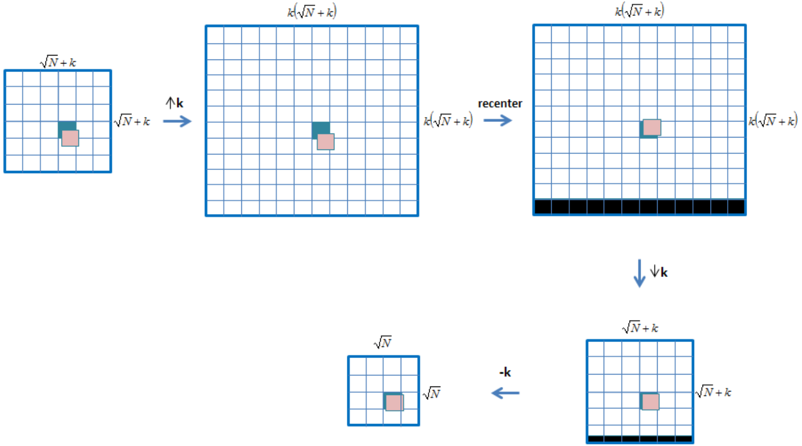
$$\mathbf{V}_w = \sum_j w_j \mathbf{V}_j = \left(\sum_j w_j \mathbf{S}_j \mathbf{I}_{V_0}\right) \otimes \mathbf{f}$$

$$w_j^* = \arg\min_{w_j} \ \frac{1}{2} \|\mathbf{V}_w - \hat{\mathbf{g}}\|_2^2$$



Data preprocessing

- Recentering the patchs



Outline

1. Introduction
2. Problem definition
3. Data preprocessing
4. Reconstruction
5. Noise estimation
6. Results
7. Future works

Reconstruction

- Stray light estimation

$$\mathbf{V}_w = \mathbf{F}_1 \hat{\mathbf{G}}\ \mathbf{f}_2 + \mathbf{n}_1$$

$$\mathbf{f}_2^* = \arg\min_{\mathbf{u}} \|\mathbf{V}_w - \mathbf{F}_1 \hat{\mathbf{G}}\ \mathbf{u}\|_2$$

- Image reconstruction

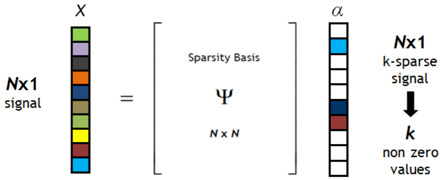
$$\mathbf{y} = \mathbf{F}_1 \mathbf{F}_2^*\ \mathbf{g} + \mathbf{n}_2$$

$$\mathbf{g}^* = \arg\min_{\mathbf{u}} \|\mathbf{y} - \mathbf{F}_1 \mathbf{F}_2^*\ \mathbf{u}\|_2$$

How do we make sure we have a unique solution?
The problems need to be regularized!!!

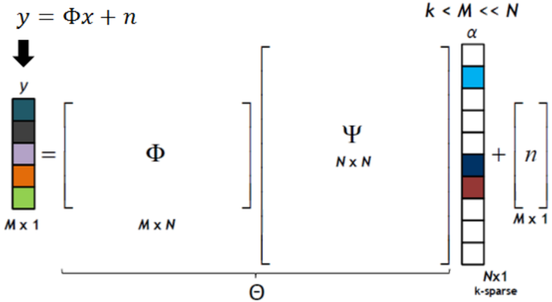
Sparsity

- A signal can be sparse or it can have a sparse representation.



- Use sparsity priors to regularize an inverse problem.

$y = \Phi x + n$



Problem regularization

- Sparsity constraint

  - Stray light is assumed to be sparse:

$$\boldsymbol{\Psi} = \mathbf{I}_N, \quad \frac{\|\mathbf{f}_2\|_1}{\|\mathbf{f}_2\|_2} \ll \sqrt{N}.$$

  - Image $\mathbf{g}$ is assumed to have a sparse representation in a wavelet basis $\boldsymbol{\Psi}$:

$$\mathbf{g} = \boldsymbol{\Psi}\boldsymbol{\alpha}, \quad \frac{\|\boldsymbol{\alpha}\|_1}{\|\boldsymbol{\alpha}\|_2} \ll \sqrt{N}.$$

  - Promotiong sparsity by minimizing the ℓ_1-norm.

- Positivity constraint

  - Both stray light and image are observations and thus are positive.

  - Promoting positivity by minimizing the convex indicator function $\imath_{\mathcal{P}}$ onto the convex set $\mathcal{P} = \{\mathbf{v} : \mathbf{v} \succeq 0\}$.

Reconstruction problem

- Stray light estimation

$$\mathbf{V}_w = \mathbf{F}_1\hat{\mathbf{G}}\ \mathbf{f}_2 + \mathbf{n}_1$$
$$\mathbf{f}_2^* = \arg\min_{\mathbf{u}} \|\mathbf{u}\|_1 \ \text{ s.t. } \ \|\mathbf{V}_w - \mathbf{F}_1\hat{\mathbf{G}}\ \mathbf{u}\|_2 \leqslant \varepsilon_1,\ \ \mathbf{u} \succeq 0$$

$$\mathbf{f}_2^* = \arg\min_{\mathbf{u}} \ \|\mathbf{u}\|_1 + \imath_{\mathcal{C}}(\mathbf{F}_1\hat{\mathbf{G}}\ \mathbf{u}) + \imath_{\mathcal{P}}(\mathbf{u})$$

$$\mathcal{C} = \{\mathbf{v} : \|\mathbf{V}_w - \mathbf{v}\|_2 \leqslant \varepsilon_1\}$$

- Image reconstruction

$$\mathbf{y} = \mathbf{F}_1\mathbf{F}_2^*\ \mathbf{g} + \mathbf{n}_2$$
$$\boldsymbol{\alpha}^* = \arg\min_{\mathbf{u}} \|\mathbf{u}\|_1 \ \text{ s.t. } \ \|\mathbf{y} - \mathbf{F}_1\mathbf{F}_2^*\boldsymbol{\Psi}\ \mathbf{u}\|_2 \leqslant \varepsilon_2, \boldsymbol{\Psi}\mathbf{u} \succeq 0$$

$$\boldsymbol{\alpha}^* = \arg\min_{\mathbf{u}} \ \|\mathbf{u}\|_1 + \imath_{\mathcal{D}}(\mathbf{F}_1\mathbf{F}_2^*\boldsymbol{\Psi}\ \mathbf{u}) + \imath_{\mathcal{P}}(\boldsymbol{\Psi}\mathbf{u})$$
$$\mathbf{g}^* = \boldsymbol{\Psi}\boldsymbol{\alpha}^*$$

$$\mathcal{D} = \{\mathbf{v} : \|\mathbf{y} - \mathbf{v}\|_2 \leqslant \varepsilon_2\}$$

Reconstruction algorithm

- Chambolle-Pock Algorithm [1]

$$\min_{\mathbf{x}\in X} \; F(\mathbf{Kx}) + G(\mathbf{x})$$

$$\begin{cases} \mathbf{v}_{k+1} & = \operatorname{prox}_{\sigma F^*}(\mathbf{v}_k + \sigma \mathbf{K}\mathbf{u}_k) \\ \mathbf{x}_{k+1} & = \operatorname{prox}_{\tau G}(\mathbf{x}_k - \tau \mathbf{K}^* \mathbf{v}_{k+1}) \\ \mathbf{u}_{k+1} & = \mathbf{x}_{k+1} + \vartheta(\mathbf{x}_{k+1} - \mathbf{x}_k) \end{cases}$$

- Proximal Operators [2]

$$\operatorname{prox}_{\lambda f} \mathbf{z} = \arg\min_{\mathbf{x}} \lambda f(\mathbf{x}) + \tfrac{1}{2}\|\mathbf{z} - \mathbf{x}\|^2$$

equivalent to gradient descent for f differentiable $(\mathbf{x}_k - \lambda \nabla f(\mathbf{x}_k))$

$$f(\mathbf{x}) = \|\mathbf{x}\|_1 \quad \Rightarrow \quad \operatorname{prox}_f \mathbf{z} = \operatorname{SoftTh}(\mathbf{z})$$
$$f(\mathbf{x}) = \imath_{\mathcal{R}}(\mathbf{x}) \quad \Rightarrow \quad \operatorname{prox}_f \mathbf{z} = \operatorname{proj}_{\mathcal{R}}(\mathbf{z})$$

[1] A. Chambolle and T. Pock. *A first-order primal-dual algorithm for convex problems with applications to imaging*. Journal of Mathematical Imaging and Vision **40**(1), 120–145. 2011.
[2] P. L. Combettes and J. C. Pesquet. *Proximal splitting methods in signal processing*. Fixed-Point Algorithms for Inverse Problems in Science and Engineering, 185-212. 2011.

Reconstruction algorithm

$$\mathbf{f}_2^* = \arg\min_{\mathbf{u}\in\mathbb{R}^N} \ \|\mathbf{u}\|_1 + \imath_{\mathcal{C}}(\mathbf{F}_1\hat{\mathbf{G}}\ \mathbf{u}) + \imath_{\mathcal{P}}(\mathbf{u})$$

$$\boldsymbol{\alpha}^* = \arg\min_{\mathbf{u}\in\mathbb{R}^N} \ \|\mathbf{u}\|_1 + \imath_{\mathcal{D}}(\mathbf{F}_1\mathbf{F}_2^*\boldsymbol{\Psi}\ \mathbf{u}) + \imath_{\mathcal{P}}(\boldsymbol{\Psi}\mathbf{u})$$

$$\mathbf{g}^* = \boldsymbol{\Psi}\boldsymbol{\alpha}^*$$

Chambolle-Pock algorithm

$$\min_{\mathbf{x}\in X} \ F(\mathbf{K}\mathbf{x}) + G(\mathbf{x})$$

Chambolle-Pock algorithm in Product-Space Optimization [3]

$$\min_{\mathbf{t}=(\mathbf{t}_1,\mathbf{t}_2)\in\mathbb{R}^{2N}} \ F_1(\mathbf{K}_1\mathbf{t}_1) + F_2(\mathbf{K}_2\mathbf{t}_2) + H(\mathbf{t}_1) + \imath_{\Pi_{1,2}}(\mathbf{t})$$

$$\Pi_{1,2} = \{\mathbf{t} : \mathbf{t}_1 = \mathbf{t}_2\}$$

[3] A. González, L. Jacques, C. De Vleeschouwer and P. Antoine. *Compressive Optical Deflectometric Tomography: A Constrained Total-Variation Minimization Approach*. Submitted in the Journal of Inverse Problem and Imaging on September 4, 2012 (arXiv:1209.0654).

Outline

1 Introduction

2 Problem definition

3 Data preprocessing

4 Reconstruction

5 Noise estimation

6 Results

7 Future works

Noise estimation

- Image Reconstruction

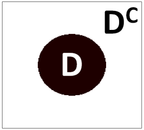
$$\mathbf{y} = \mathbf{f}_1 \otimes \mathbf{f}_2^* \otimes \mathbf{g} + \mathbf{n}_2$$

$\mathbf{n}_2$: some observation noise.

- Stray light estimation

$$\begin{aligned}
\mathbf{V}_w &= \mathbf{f}_1 \otimes \hat{\mathbf{g}} \otimes \mathbf{f}_2 + \mathbf{n}_1 \\
\mathbf{V}_w &= \mathbf{f}_1 \otimes \hat{\mathbf{g}}_q \otimes \mathbf{f}_2 \\
&= \mathbf{f}_1 \otimes (\hat{\mathbf{g}} + \mathbf{q}) \otimes \mathbf{f}_2 \\
&= \mathbf{f}_1 \otimes \hat{\mathbf{g}} \otimes \mathbf{f}_2 + \mathbf{f}_1 \otimes \mathbf{q} \otimes \mathbf{f}_2
\end{aligned}$$

$$q_i = \begin{cases} q_{in} \sim \mathcal{N}(\mu_1, \sigma_1^2) \text{ if i} \in \mathrm{D} \\ q_{out} \sim \mathcal{N}(\mu_2, \sigma_2^2) \text{ if i} \in \mathrm{D^C} \end{cases}$$



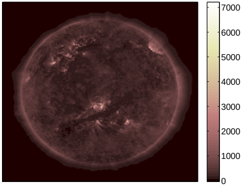
$$\mathbf{n}_1 = \mathbf{f}_1 \otimes \mathbf{q} \otimes \mathbf{f}_2$$

$+$ some observation noise and modeling noise.

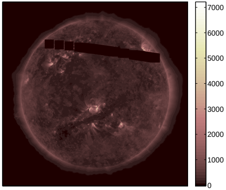
$$\begin{aligned}
\|\mathbf{n}_1\|_2 \leqslant \varepsilon_1 &= \|\mathbf{f}_1 \otimes \mathbf{q} \otimes \mathbf{f}_2\|_2 \\
&\approx \sigma_2 \|\mathbf{f}_1 \otimes \mathbf{f}_2\|_2
\end{aligned}$$

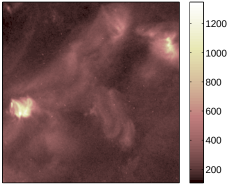
Variance estimation

$\mathbf{I}_0, \ \mathbf{I} = \mathbf{I}_0 \otimes \mathbf{f}$



$\mathbf{I}_j = \mathbf{S}_j \mathbf{I} = \mathbf{S}_j(\mathbf{I}_0 \otimes \mathbf{f}) = (\mathbf{S}_j \mathbf{I}_0) \otimes \mathbf{f}$

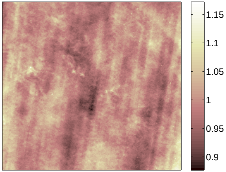




Optimized weighting

$$\mathbf{I}_w = \sum_j w_j \mathbf{I}_j = \left(\sum_j w_j \mathbf{S}_j \mathbf{I}_0\right) \otimes \mathbf{f}$$

$$w_j^* = \arg\min_{w_j} \ \frac{1}{2} \|\mathbf{I}_w - \mathbf{1}_N\|_2^2$$



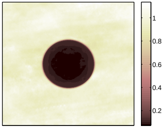
180 patchs $\Rightarrow \ \sigma_2 = 0.0332$

Outline

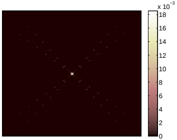
1. Introduction
2. Problem definition
3. Data preprocessing
4. Reconstruction
5. Noise estimation
6. **Results**
7. Future works

Stray light estimation

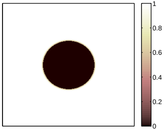
$\mathbf{V}_w$



$=$ $\mathbf{f}_1$



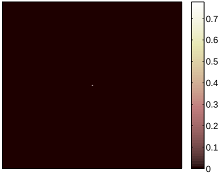
$\otimes$ $\hat{\mathbf{g}}$



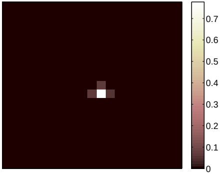
$\otimes$ $\mathbf{f}_2$ **?** $+ \ \mathbf{n}_1$

- Estimated $\mathbf{f}_2$: RSNR $=$ 13 dB. 26'. 66000 iterations

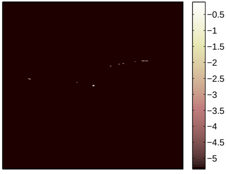
$\mathbf{f}_2^*$



$\mathbf{f}_2^*$ zoomed



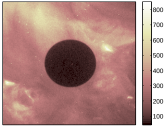
$\log_{10}(\mathbf{f}_2^*)$



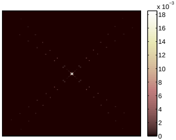
Results: Actual Scenario

Image reconstruction, assuming no stray light

$\mathbf{y}$ $\qquad$ $\mathbf{f}_1$ $\qquad$ $\mathbf{g}$



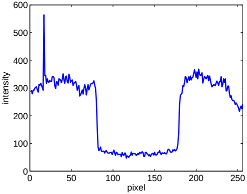
$=$



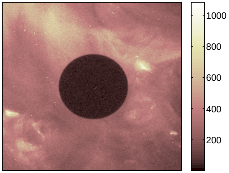
$\otimes$ **?** $+$ $\mathbf{n}_2$

- Estimated $\mathbf{g}$: 23'. 50000 iterations

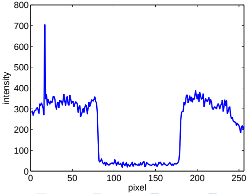
$\mathbf{y}$, px $= 128$



$\mathbf{g}^*$

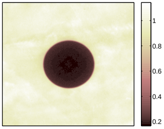


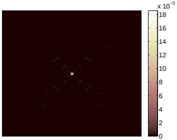
$\mathbf{g}^*$, px $= 128$

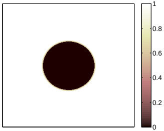


Results: Actual Scenario

Stray light estimation

$\mathbf{V}_w$ 

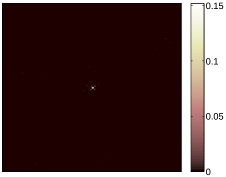
$=$ $\mathbf{f}_1$ 

$\otimes$ $\hat{\mathbf{g}}$ 

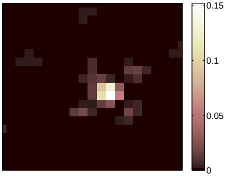
$\otimes$ $\mathbf{f}_2$ **?** $+$ $\mathbf{n}_1$

- Estimated $\mathbf{f}_2$: 28'. 61000 iterations

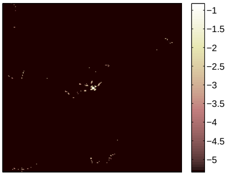
$\mathbf{f}_2^*$



$\mathbf{f}_2^*$ zoomed



$\log_{10}(\mathbf{f}_2^*)$

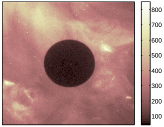


Results: Actual Scenario

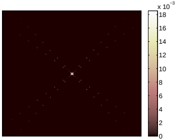
Image reconstruction, with stray light correction

$$\mathbf{y} = \mathbf{f}_1 \otimes \mathbf{f}_2^* \otimes \mathbf{g} + \mathbf{n}_2$$

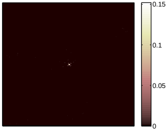
$\mathbf{y}$



$\mathbf{f}_1$



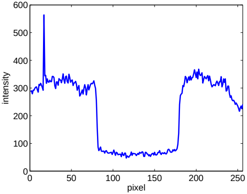
$\mathbf{f}_2^*$



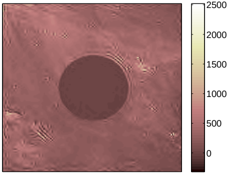
$\mathbf{g}$: ?

- Estimated $\mathbf{g}$: 6'. 10000 iterations

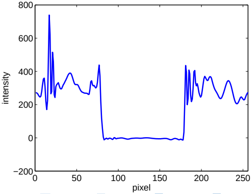
$\mathbf{y}$, px = 128



$\mathbf{g}^*$



$\mathbf{g}^*$, px = 128



Outline

1 Introduction

2 Problem definition

3 Data preprocessing

4 Reconstruction

5 Noise estimation

6 Results

7 Future works

Future works

- Improve noise estimation, including observation noise and modeling noise.
  Reduce existing artifacts in the stray light estimation.

- Stabilize image reconstruction.

- Minimize both f_2 and g simultaneously.
  $\Rightarrow$ Non-convex optimization problem.

Thanks for your attention!!