

A DECONVOLUTION PROBLEM IN ASTRONOMY

Image and Signal Processing Seminars

Adriana González

Joint work with Prof. Laurent Jacques and in collaboration with Dr. Véronique Delouille from the Royal Observatory of Belgium.

UCL/ICTEAM/ELEN

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Outline

- 1 Introduction
- 2 Problem definition
- 3 Data preprocessing
- 4 Reconstruction
- 5 Noise estimation
- 6 Results
- 7 Future works

Outline

1 Introduction

2 Problem definition

3 Data preprocessing

4 Reconstruction

5 Noise estimation

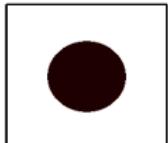
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Imaging distortions

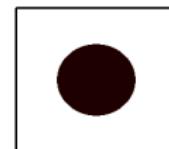
Ideally

Observation



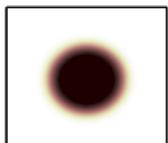
=

Image



But actually

Observation



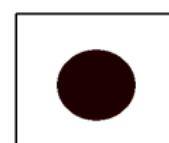
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PSF



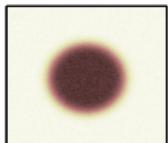
\otimes

Image



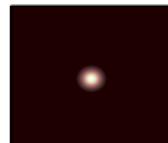
Moreover

Observation



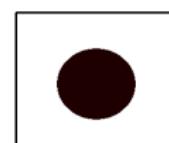
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PSF



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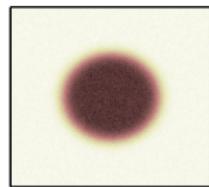
Image



+ noise

Blind deconvolution problem

Observation

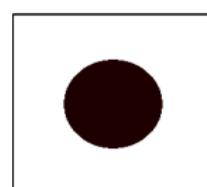


=



PSF

\otimes



Image

+ n

?

?

⇓

What we need

Outline

1 Introduction

2 Problem definition

3 Data preprocessing

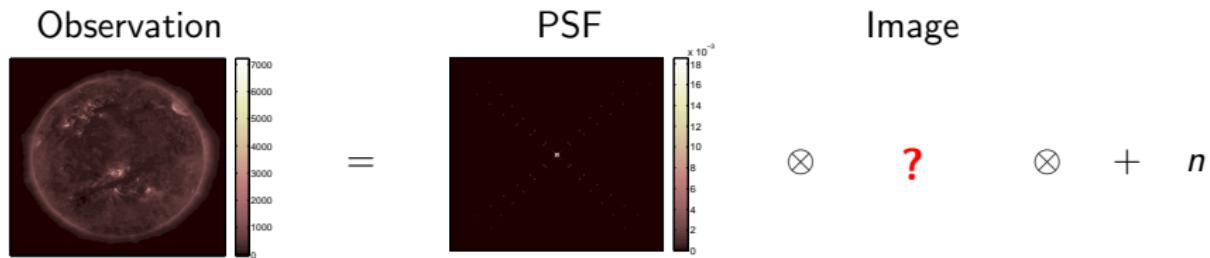
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Problem definition



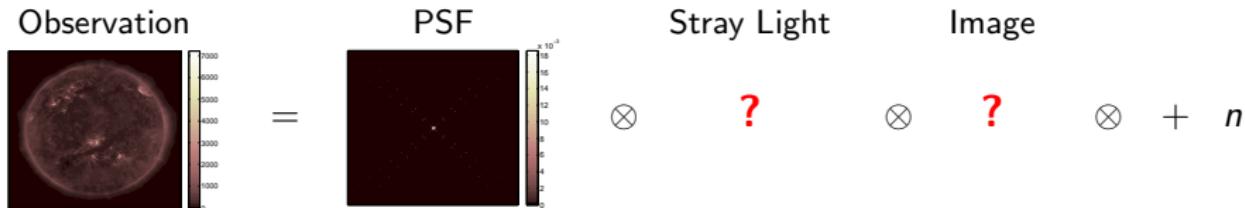
Easy to solve!!! But...
Observations are also affected by stray light.

Problem definition

- Stray light

⇒ undesired light in the optical system.

⇒ modeled as an additional filter convolving the image.

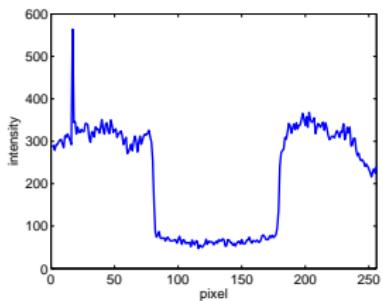
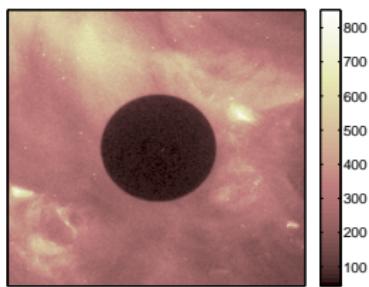
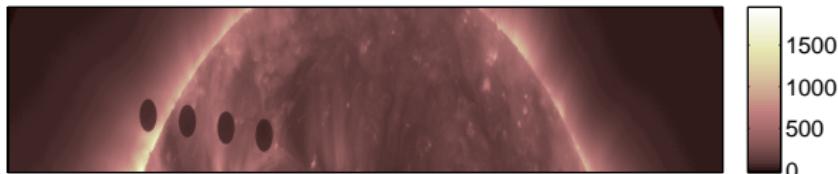


Problem

Stray light correction to study low intensity regions in the Sun.

Available information

- Venus transit: June 6, 2012.



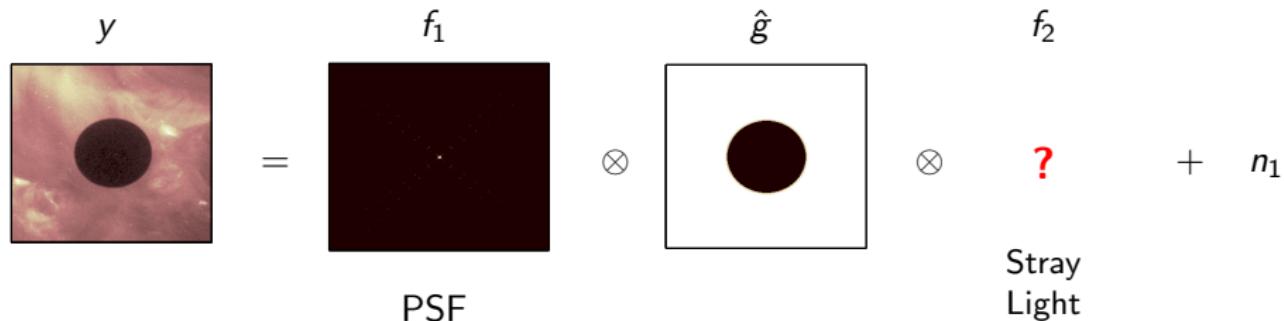
Prior information

Venus is a **uniformly black disk** with a known diameter.

Redefining the problem

1. Stray light estimation: Estimating f_2

$$y = f_1 \otimes \hat{g} \otimes f_2 + n_1 \quad \Rightarrow \quad \mathbf{y} = \mathbf{F}_1 \hat{\mathbf{G}} \mathbf{f}_2 + \mathbf{n}_1$$



2. Stray light correction: Estimating g

$$y = f_1 \otimes f_2 \otimes g + n_2 \quad \Rightarrow \quad \mathbf{y} = \mathbf{F}_1 \mathbf{F}_2 \mathbf{g} + \mathbf{n}_2$$

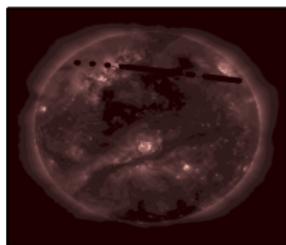
$$\mathbf{y} = \mathbf{F} \mathbf{g} + \mathbf{n}_2$$

Outline

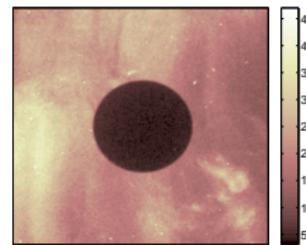
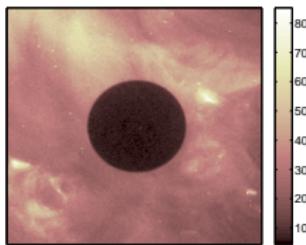
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Data preprocessing

$$\mathbf{I}_V = \mathbf{I}_{V_0} \otimes \mathbf{f}, \quad \mathbf{f} = \mathbf{f}_1 \otimes \mathbf{f}_2$$



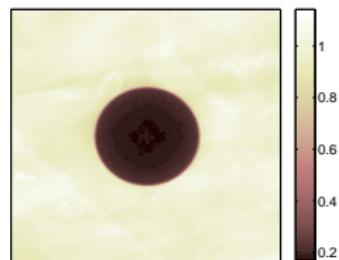
$$\mathbf{V}_j = \mathbf{S}_j \mathbf{I}_V = \mathbf{S}_j (\mathbf{I}_{V_0} \otimes \mathbf{f}) = (\mathbf{S}_j \mathbf{I}_{V_0}) \otimes \mathbf{f}$$



Optimized weighting

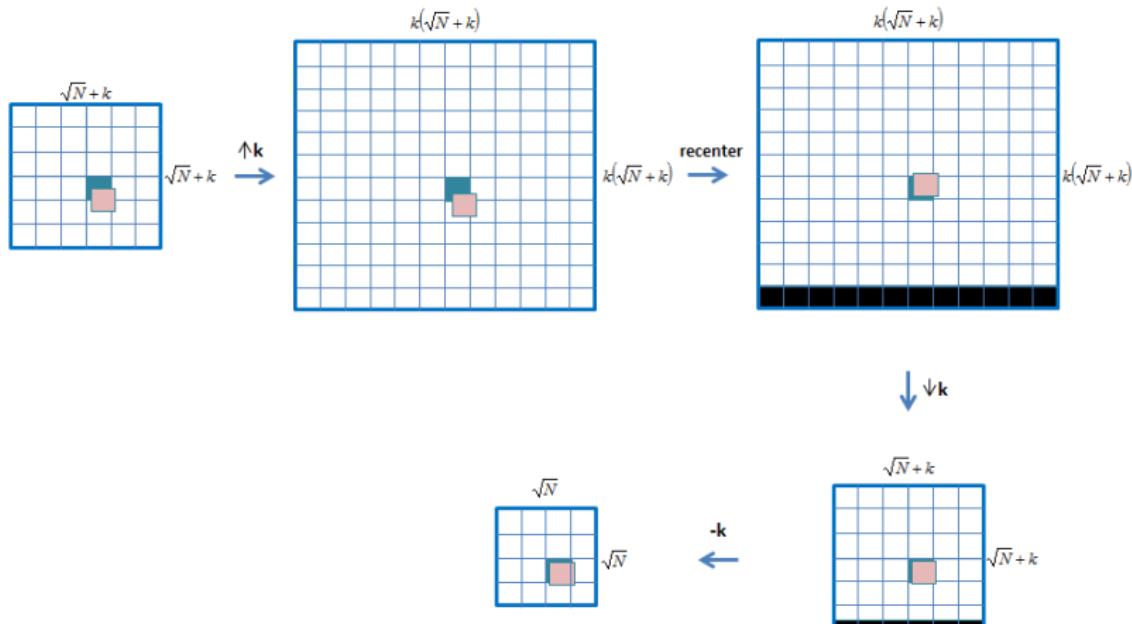
$$\mathbf{V}_w = \sum_j w_j \mathbf{V}_j = (\sum_j w_j \mathbf{S}_j \mathbf{I}_{V_0}) \otimes \mathbf{f}$$

$$w_j^* = \arg \min_{w_j} \frac{1}{2} \|\mathbf{V}_w - \hat{\mathbf{g}}\|_2^2$$



Data preprocessing

- Recentering the patches



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Reconstruction

- Stray light estimation

$$\mathbf{V}_w = \mathbf{F}_1 \hat{\mathbf{G}} \mathbf{f}_2 + \mathbf{n}_1$$

$$\mathbf{f}_2^* = \arg \min_{\mathbf{u}} \|\mathbf{V}_w - \mathbf{F}_1 \hat{\mathbf{G}} \mathbf{u}\|_2$$

- Image reconstruction

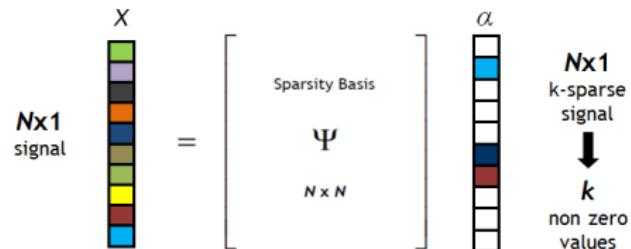
$$\mathbf{y} = \mathbf{F}_1 \mathbf{F}_2^* \mathbf{g} + \mathbf{n}_2$$

$$\mathbf{g}^* = \arg \min_{\mathbf{u}} \|\mathbf{y} - \mathbf{F}_1 \mathbf{F}_2^* \mathbf{u}\|_2$$

**How do we make sure we have a unique solution?
The problems need to be regularized!!!**

Sparsity

- A signal can be sparse or it can have a sparse representation.



- Use sparsity priors to regularize an inverse problem.

$$y = \Phi x + n$$

The diagram illustrates the inverse problem $y = \Phi x + n$. On the left, a vector y (M x 1) is shown with colored segments (dark blue, grey, purple, orange, green). An equals sign follows this vector. To the right is a matrix Φ (M x N). To the right of the matrix is a vector Ψ (N x N). Below the matrix Φ is a bracket labeled Θ , indicating that Φ and Ψ are part of the system Θ . To the right of the Ψ matrix is a vector α (N x 1), which is described as "N_x1 k-sparse". To the right of the α vector is a plus sign, followed by a vector n (M x 1).

Problem regularization

- Sparsity constraint

- Stray light is assumed to be sparse:

$$\Psi = \mathbf{I}_N, \quad \frac{\|\mathbf{f}_2\|_1}{\|\mathbf{f}_2\|_2} \ll \sqrt{N}.$$

- Image \mathbf{g} is assumed to have a sparse representation in a wavelet basis Ψ :

$$\mathbf{g} = \Psi \boldsymbol{\alpha}, \quad \frac{\|\boldsymbol{\alpha}\|_1}{\|\boldsymbol{\alpha}\|_2} \ll \sqrt{N}.$$

- Promotioong sparsity by minimizing the ℓ_1 -norm.

- Positivity constraint

- Both stray light and image are observations and thus are positive.
 - Promoting positivity by minimizing the convex indicator function $\iota_{\mathcal{P}}$ onto the convex set $\mathcal{P} = \{\mathbf{v} : \mathbf{v} \succeq 0\}$.

Reconstruction problem

- Stray light estimation

$$\mathbf{V}_w = \mathbf{F}_1 \hat{\mathbf{G}} \mathbf{f}_2 + \mathbf{n}_1$$
$$\mathbf{f}_2^* = \arg \min_{\mathbf{u}} \|\mathbf{u}\|_1 \text{ s.t. } \|\mathbf{V}_w - \mathbf{F}_1 \hat{\mathbf{G}} \mathbf{u}\|_2 \leq \varepsilon_1, \mathbf{u} \succeq 0$$

$$\mathbf{f}_2^* = \arg \min_{\mathbf{u}} \|\mathbf{u}\|_1 + \iota_{\mathcal{C}}(\mathbf{F}_1 \hat{\mathbf{G}} \mathbf{u}) + \iota_{\mathcal{P}}(\mathbf{u})$$

$$\mathcal{C} = \{\mathbf{v} : \|\mathbf{V}_w - \mathbf{v}\|_2 \leq \varepsilon_1\}$$

- Image reconstruction

$$\mathbf{y} = \mathbf{F}_1 \mathbf{F}_2^* \mathbf{g} + \mathbf{n}_2$$
$$\alpha^* = \arg \min_{\mathbf{u}} \|\mathbf{u}\|_1 \text{ s.t. } \|\mathbf{y} - \mathbf{F}_1 \mathbf{F}_2^* \Psi \mathbf{u}\|_2 \leq \varepsilon_2, \Psi \mathbf{u} \succeq 0$$

$$\alpha^* = \arg \min_{\mathbf{u}} \|\mathbf{u}\|_1 + \iota_{\mathcal{D}}(\mathbf{F}_1 \mathbf{F}_2^* \Psi \mathbf{u}) + \iota_{\mathcal{P}}(\Psi \mathbf{u})$$

$$\mathbf{g}^* = \Psi \alpha^*$$

$$\mathcal{D} = \{\mathbf{v} : \|\mathbf{y} - \mathbf{v}\|_2 \leq \varepsilon_2\}$$

Reconstruction algorithm

- Chambolle-Pock Algorithm [1]

$$\min_{\mathbf{x} \in X} F(\mathbf{Kx}) + G(\mathbf{x})$$

$$\begin{cases} \mathbf{v}_{k+1} &= \text{prox}_{\sigma F^*}(\mathbf{v}_k + \sigma \mathbf{Ku}_k) \\ \mathbf{x}_{k+1} &= \text{prox}_{\tau G}(\mathbf{x}_k - \tau \mathbf{K}^* \mathbf{v}_{k+1}) \\ \mathbf{u}_{k+1} &= \mathbf{x}_{k+1} + \vartheta(\mathbf{x}_{k+1} - \mathbf{x}_k) \end{cases}$$

- Proximal Operators [2]

$$\text{prox}_{\lambda f} \mathbf{z} = \arg \min_{\mathbf{x}} \lambda f(\mathbf{x}) + \frac{1}{2} \|\mathbf{z} - \mathbf{x}\|^2$$

equivalent to gradient descent for f differentiable $(\mathbf{x}_k - \lambda \nabla f(\mathbf{x}_k))$

$$\begin{aligned} f(\mathbf{x}) = \|\mathbf{x}\|_1 &\Rightarrow \text{prox}_f \mathbf{z} = \text{SoftTh}(\mathbf{z}) \\ f(\mathbf{x}) = \iota_{\mathcal{R}}(\mathbf{x}) &\Rightarrow \text{prox}_f \mathbf{z} = \text{proj}_{\mathcal{R}}(\mathbf{z}) \end{aligned}$$

[1] A. Chambolle and T. Pock. A first-order primal-dual algorithm for convex problems with applications to imaging. *Journal of Mathematical Imaging and Vision* **40**(1), 120–145. 2011.

[2] P. L. Combettes and J. C. Pesquet. Proximal splitting methods in signal processing. *Fixed-Point Algorithms for Inverse Problems in Science and Engineering*, 185–212. 2011.

Reconstruction algorithm

$$\mathbf{f}_2^* = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \|\mathbf{u}\|_1 + \varphi_C(\mathbf{F}_1 \hat{\mathbf{G}} \mathbf{u}) + \varphi_P(\mathbf{u})$$

$$\alpha^* = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \|\mathbf{u}\|_1 + \varphi_D(\mathbf{F}_1 \mathbf{F}_2^* \Psi \mathbf{u}) + \varphi_P(\Psi \mathbf{u})$$

$$\mathbf{g}^* = \Psi \alpha^*$$

Chambolle-Pock algorithm

$$\min_{\mathbf{x} \in X} F(\mathbf{Kx}) + G(\mathbf{x})$$

Chambolle-Pock algorithm in Product-Space Optimization [3]

$$\min_{\mathbf{t}=(\mathbf{t}_1, \mathbf{t}_2) \in \mathbb{R}^{2N}} F_1(\mathbf{K}_1 \mathbf{t}_1) + F_2(\mathbf{K}_2 \mathbf{t}_2) + H(\mathbf{t}_1) + \varphi_{\Pi_{1,2}}(\mathbf{t})$$

$$\Pi_{1,2} = \{\mathbf{t} : \mathbf{t}_1 = \mathbf{t}_2\}$$

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Noise estimation

- Image Reconstruction

$$\mathbf{y} = \mathbf{f}_1 \otimes \mathbf{f}_2^* \otimes \mathbf{g} + \mathbf{n}_2$$

\mathbf{n}_2 : some observation noise.

- Stray light estimation

$$\mathbf{V}_w = \mathbf{f}_1 \otimes \hat{\mathbf{g}} \otimes \mathbf{f}_2 + \mathbf{n}_1$$

$$\mathbf{V}_w = \mathbf{f}_1 \otimes \hat{\mathbf{g}}_q \otimes \mathbf{f}_2$$

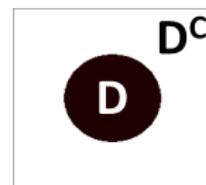
$$= \mathbf{f}_1 \otimes (\hat{\mathbf{g}} + \mathbf{q}) \otimes \mathbf{f}_2$$

$$= \mathbf{f}_1 \otimes \hat{\mathbf{g}} \otimes \mathbf{f}_2 + \mathbf{f}_1 \otimes \mathbf{q} \otimes \mathbf{f}_2$$

$$\mathbf{n}_1 = \mathbf{f}_1 \otimes \mathbf{q} \otimes \mathbf{f}_2$$

+ some observation noise and
modeling noise.

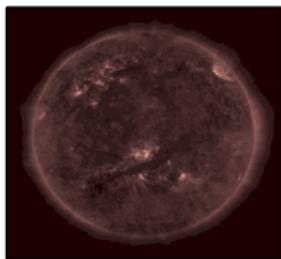
$$q_i = \begin{cases} q_{in} \sim \mathcal{N}(\mu_1, \sigma_1^2) & \text{if } i \in D \\ q_{out} \sim \mathcal{N}(\mu_2, \sigma_2^2) & \text{if } i \in D^C \end{cases}$$



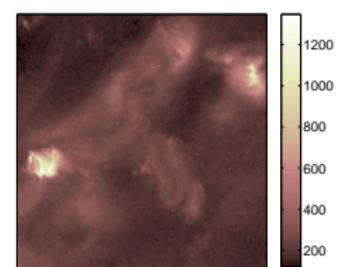
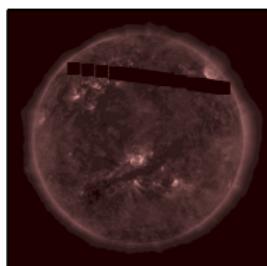
$$\begin{aligned}\|\mathbf{n}_1\|_2 &\leq \varepsilon_1 = \|\mathbf{f}_1 \otimes \mathbf{q} \otimes \mathbf{f}_2\|_2 \\ &\approx \sigma_2 \|\mathbf{f}_1 \otimes \mathbf{f}_2\|_2\end{aligned}$$

Variance estimation

$$\mathbf{I}_0, \mathbf{I} = \mathbf{I}_0 \otimes \mathbf{f}$$



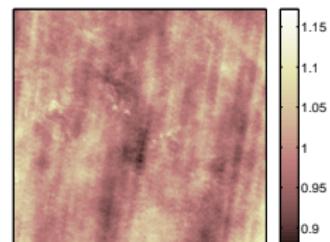
$$\mathbf{I}_j = \mathbf{S}_j \mathbf{I} = \mathbf{S}_j (\mathbf{I}_0 \otimes \mathbf{f}) = (\mathbf{S}_j \mathbf{I}_0) \otimes \mathbf{f}$$



Optimized weighting

$$\mathbf{I}_w = \sum_j w_j \mathbf{I}_j = (\sum_j w_j \mathbf{S}_j \mathbf{I}_0) \otimes \mathbf{f}$$

$$w_j^* = \arg \min_{w_j} \frac{1}{2} \|\mathbf{I}_w - \mathbf{1}_N\|_2^2$$



180 patches $\Rightarrow \sigma_2 = 0.0332$

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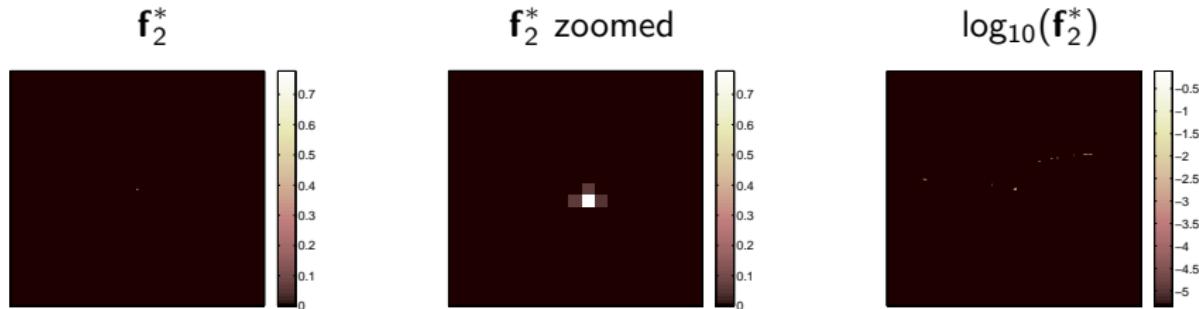
Results: Synthetic Scenario

Stray light estimation

$$\mathbf{V}_w = \mathbf{f}_1 \otimes \hat{\mathbf{g}} + \mathbf{f}_2 \otimes ? + \mathbf{n}_1$$

The diagram illustrates the mathematical model for stray light estimation. It shows the observed image \mathbf{V}_w as a yellow square with a black circular hole, accompanied by its color bar ranging from 0 to 1. This is equated to the sum of three terms: \mathbf{f}_1 (a dark image with scattered white noise) multiplied by the estimated stray light $\hat{\mathbf{g}}$ (a yellow square with a black circle, color bar 0-1), plus \mathbf{f}_2 (another dark image with scattered white noise) multiplied by an unknown stray light component (represented by a red question mark), plus a noise term \mathbf{n}_1 .

- Estimated \mathbf{f}_2 : RSNR = 13 dB. 26'. 66000 iterations



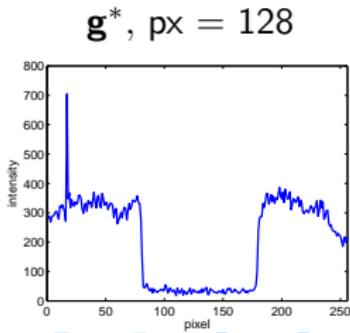
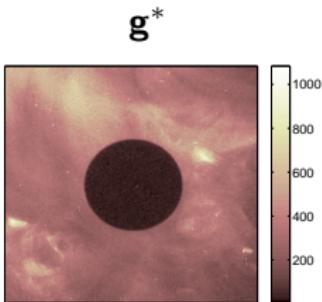
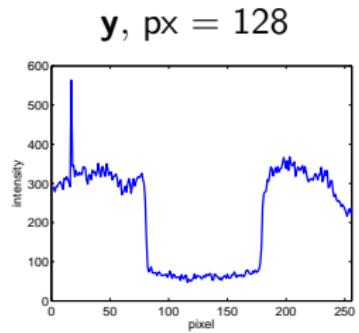
Results: Actual Scenario

Image reconstruction, assuming no stray light

$$\mathbf{y} = \mathbf{f}_1 \otimes ? + \mathbf{n}_2$$

The equation illustrates the model for image reconstruction. The observed image \mathbf{y} is equal to the product of the operator \mathbf{f}_1 and the unknown image \mathbf{g} , plus a noise term \mathbf{n}_2 . The operator \mathbf{f}_1 is shown as a dark matrix with a few scattered white pixels, and its color bar indicates values ranging from 0 to approximately 18 units of 10^{-3} .

- Estimated \mathbf{g} : 23'. 50000 iterations



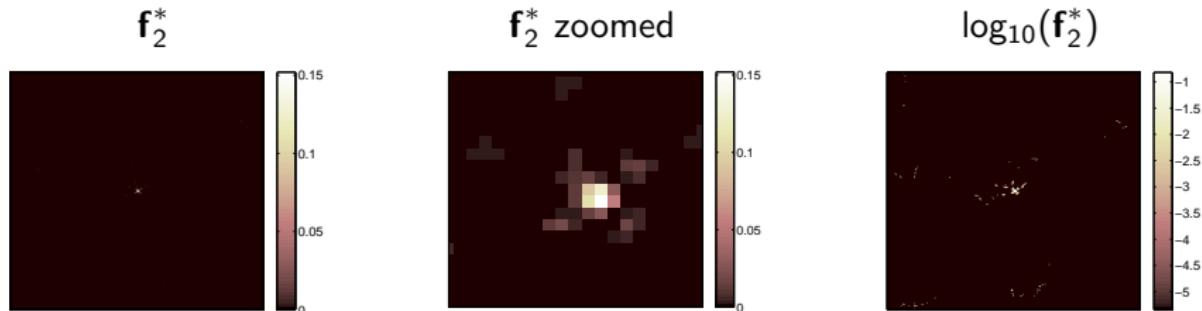
Results: Actual Scenario

Stray light estimation

$$\mathbf{V}_w = \mathbf{f}_1 \otimes \hat{\mathbf{g}} \otimes \mathbf{f}_2 + \mathbf{n}_1$$

The diagram illustrates the mathematical model for stray light estimation. It shows the decomposition of the total signal \mathbf{V}_w into its components. \mathbf{V}_w is represented by a grayscale image of a central dark circle on a textured background, with a color bar ranging from 0.2 to 1. This is equated to the product of three matrices: \mathbf{f}_1 (a dark image with scattered white dots, color bar 0 to 18), $\hat{\mathbf{g}}$ (a white image with a central black circle, color bar 0 to 1), and \mathbf{f}_2 (a dark image, color bar 0 to 1). The multiplication is indicated by the symbol \otimes . The final term \mathbf{n}_1 is represented by a red question mark.

- Estimated \mathbf{f}_2 : 28'. 61000 iterations



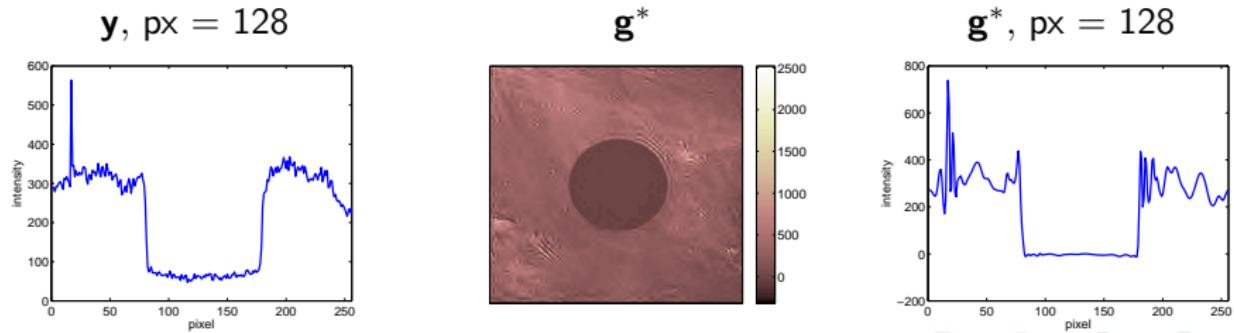
Results: Actual Scenario

Image reconstruction, with stray light correction

$$y = f_1 \otimes f_2^* \otimes ? + n_2$$

The diagram illustrates the image reconstruction process. On the left is the observed image y , which shows a central black circle against a textured background. A color bar to its right ranges from 100 to 800. In the center is the forward operator f_1 , represented by a dark matrix with a few scattered white pixels. To its right is the adjoint operator f_2^* , also a dark matrix with scattered white pixels. Between f_1 and f_2^* is a multiplication symbol (\otimes). To the right of f_2^* is another multiplication symbol (\otimes) followed by a question mark ($?$). Further to the right is a plus sign ($+$) and the noise term n_2 .

- Estimated g : 6'. 10000 iterations



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Future works

- Improve noise estimation, including observation noise and modeling noise.
 Reduce existing artifacts in the stray light estimation.
- Stabilize image reconstruction.
- Minimize both f_2 and g simultaneously.
 ⇒ Non-convex optimization problem.

Thanks for your attention!!