

Computational Imaging with Plug-and-play priors: Leverage the Power of Deep Learning

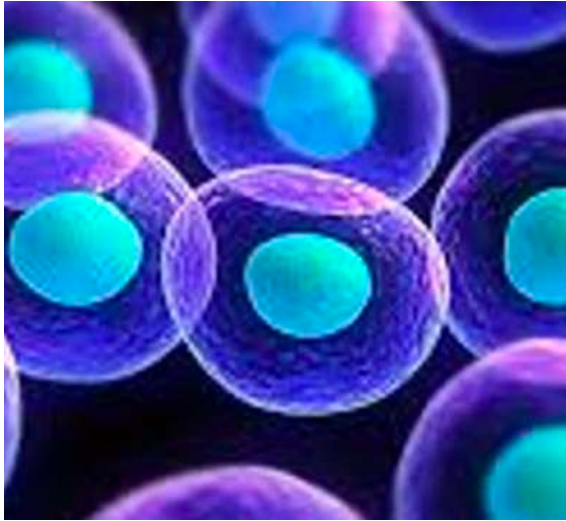
Xiaojian Xu

Supervised by Dr. Ulugbek Kamilov

Washington University in St. Louis

ISPG-Seminar – Dec 9 2020

Imaging is everywhere



imaging in microscopy



imaging in photography



imaging in astronomy

Big shift in imaging

- Imaging not by taking pictures but by computing pictures

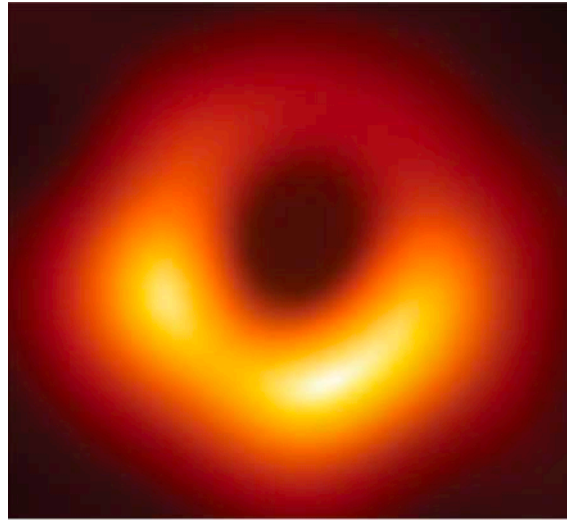


Computational imaging applications

- In computational imaging, we don't have direct access to the thing we want



imaging skeleton

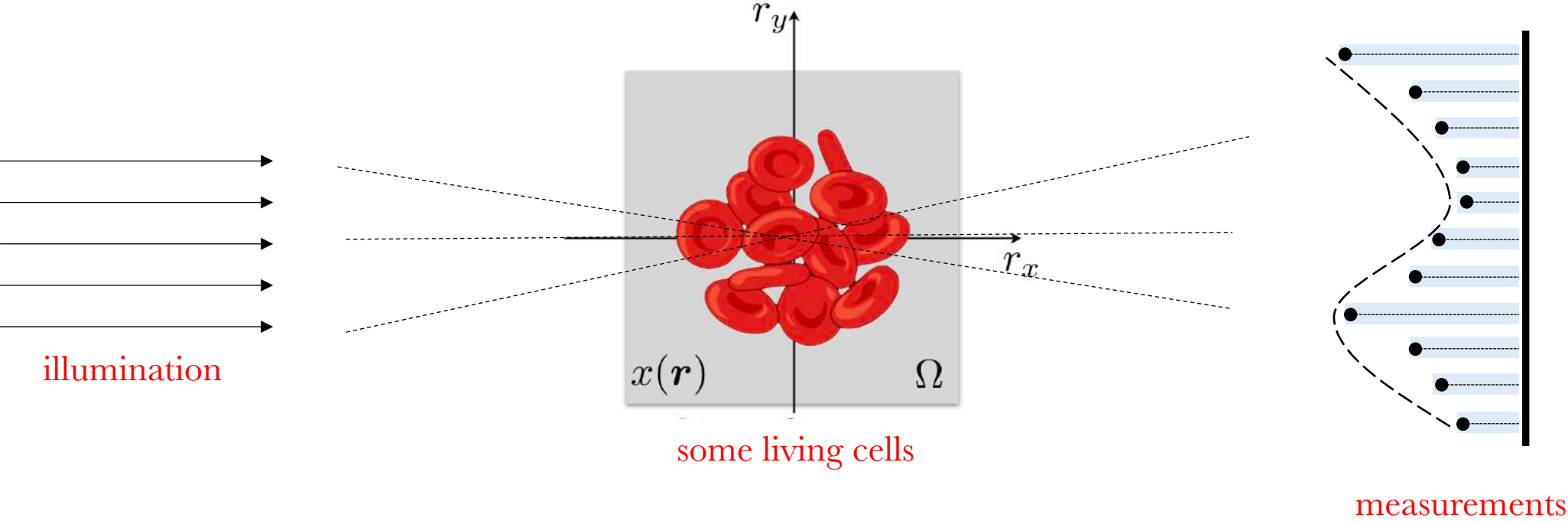


imaging blackhole

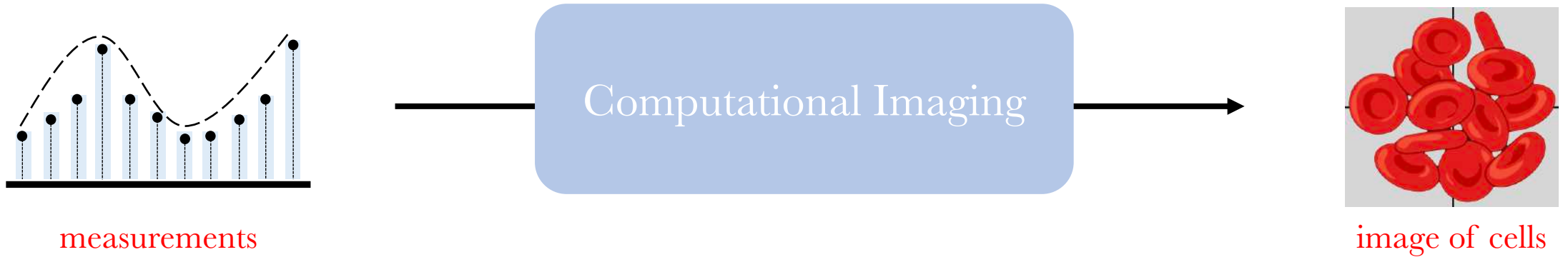


imaging infant

A simple microscopy imaging problem (Acquisition)



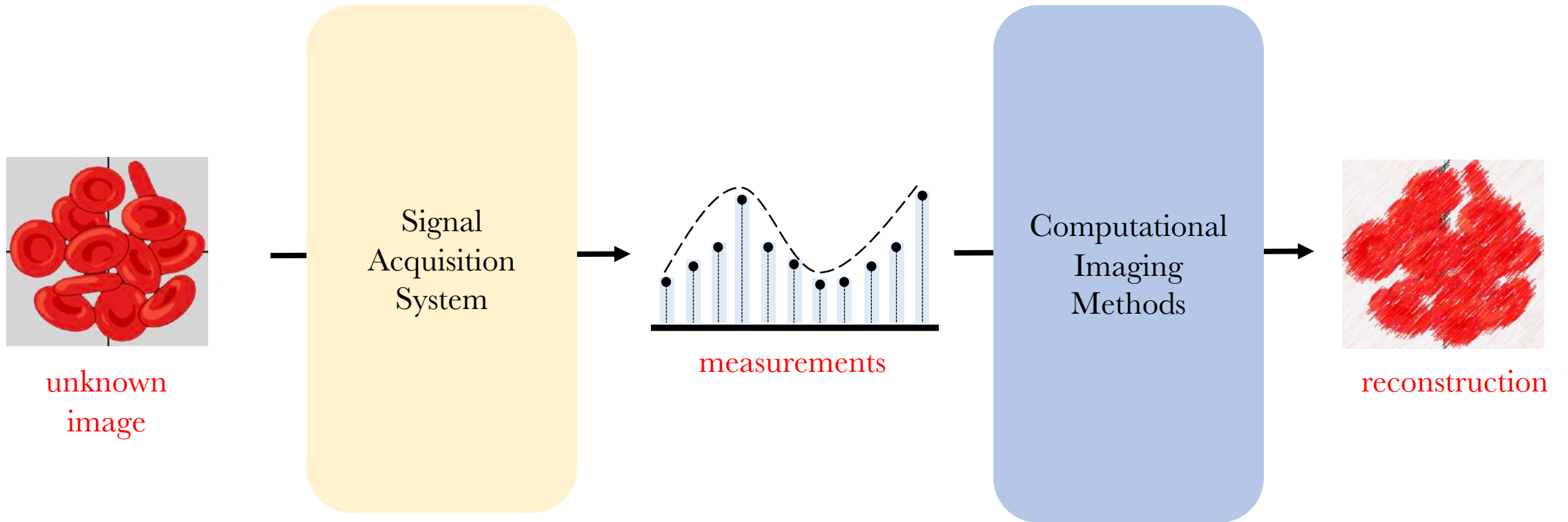
A simple microscopy imaging problem (Reconstruction)



Computational Imaging is the process of **indirectly forming images** from **measurements** using algorithms that rely on a significant amount of computing.

source: https://en.wikipedia.org/wiki/Computational_imaging

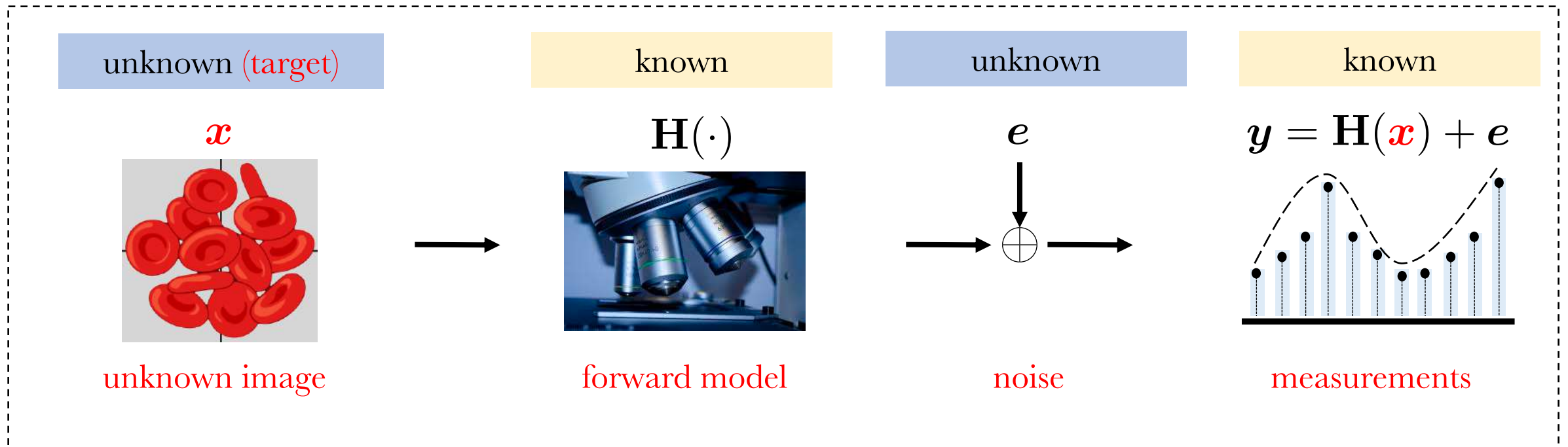
Imaging needs two procedures



Imaging as an inverse problem

Signal Acquisition Procedure

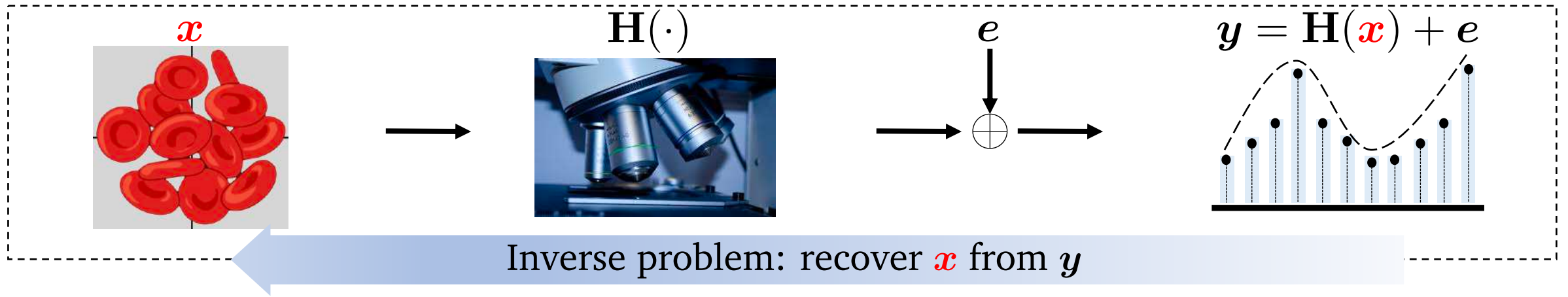
Forward problem: generate y from x



Inverse problem: recover x from y

Image Reconstruction Procedure

Imaging inverse problems are challenging



What makes imaging inverse problems challenging?

- ❖ Solution is not unique
- ❖ Measurements are noisy
- ❖ Image x can be high-dimensional

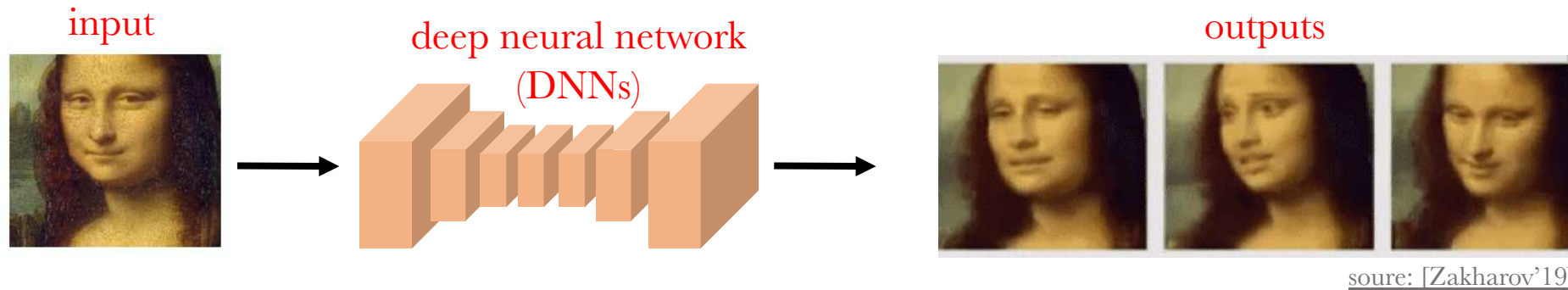
Computation imaging methods

- Formulate it as a **regularized optimization** task (model-based optimization)

$$\hat{x} = \arg \min_x \{ \overset{\text{data-fidelity}}{g(x)} + \overset{\text{prior/regularizer}}{h(x)} \}$$

example: $\hat{x} = \arg \min_x \left\{ \frac{1}{2} \|\mathbf{H}(x) - \mathbf{y}\|^2 + h(x) \right\}$

- Learn an **end-to-end mapping**



- Combining **optimization & learning!**



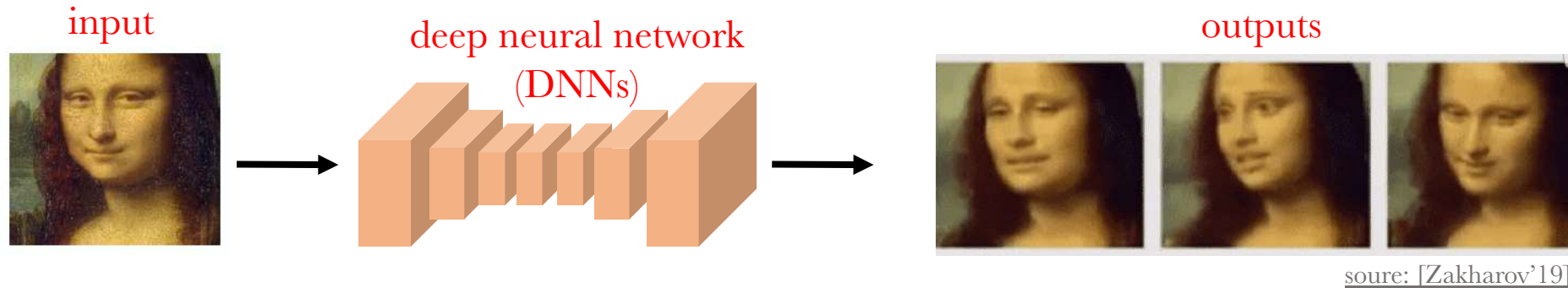
Computation imaging methods

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- Learn an **end-to-end mapping**



- Combining **optimization & learning!**



Today we will talk about

1. Introduction to Plug-and-Play Priors (PnP)

- Denoiser strength selecting challenge
- Optimization interpretation and convergence analysis challenge

2. Denoiser scaling technique

- [Xu'20(1)] X. Xu et.al. Boosting the Performance of Plug-and-Play Priors via Denoiser Scaling

3. Optimization interpretation and convergence analysis of PnP with MMSE denoisers

- [Xu'20(2)] X. Xu et.al. Provable Convergence of Plug-and-Play Priors With MMSE Denoisers,

Imaging as a regularized optimization task

- Recall our **regularized optimization task**

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \{ \overset{\text{data-fidelity}}{g(\mathbf{x})} + \overset{\text{prior/regularizer}}{h(\mathbf{x})} \}$$

Example: Fast iterative shrinkage/thresholding algorithm (**FISTA**) [Nesterov'13] & Alternating direction method of multipliers (**ADMM**) [Boyd'10]

FISTA

$$\begin{aligned} \mathbf{z}^k &\leftarrow \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1}) \\ \mathbf{x}^k &\leftarrow \text{prox}_{\gamma h}(\mathbf{z}^k) \\ \mathbf{s}^k &\leftarrow \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1}) \end{aligned}$$

ADMM

$$\begin{aligned} \mathbf{z}^k &\leftarrow \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1}) \\ \mathbf{x}^k &\leftarrow \text{prox}_{\gamma h}(\mathbf{z}^k + \mathbf{s}^{k-1}) \\ \mathbf{s}^k &\leftarrow \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k) \end{aligned}$$

Proximal algorithms

$$\hat{\mathbf{x}} = \arg \min_x \{g(\mathbf{x}) + h(\mathbf{x})\}$$

model prior

- Let's take a closer look at these two proximal algorithms

FISTA

$$\mathbf{z}^k \leftarrow \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

increase data consistency

$$\mathbf{x}^k \leftarrow \text{prox}_{\gamma h}(\mathbf{z}^k)$$

reduce noise

$$\mathbf{s}^k \leftarrow \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$

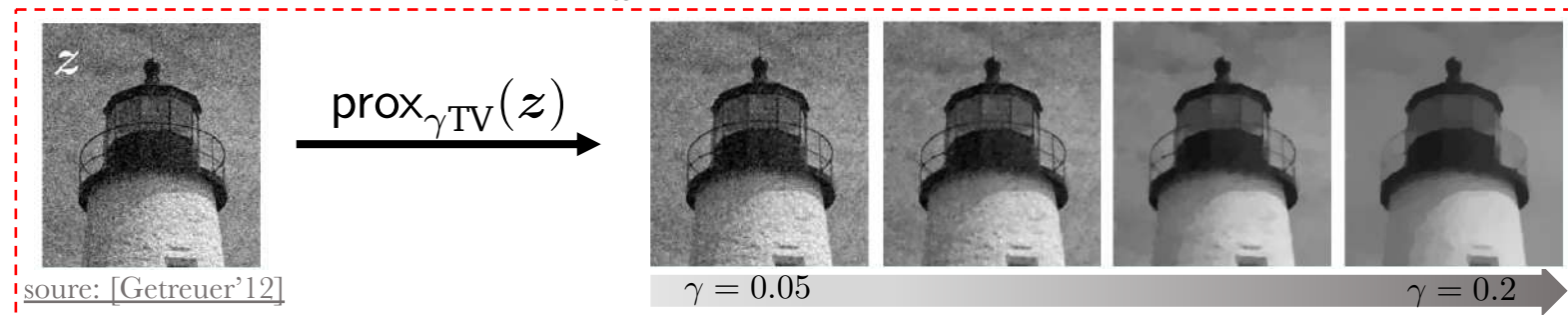
ADMM

$$\mathbf{z}^k \leftarrow \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k \leftarrow \text{prox}_{\gamma h}(\mathbf{z}^k + \mathbf{s}^{k-1})$$

$$\mathbf{s}^k \leftarrow \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

Definition: $\text{prox}_{\gamma h}(\mathbf{z}) := \arg \min_{\mathbf{x}} \left\{ \frac{1}{2} \|\mathbf{x} - \mathbf{z}\|_2^2 + \gamma h(\mathbf{x}) \right\}$ image denoiser for AWGN



Proximal algorithms

$$\hat{\mathbf{x}} = \arg \min_x \{g(\mathbf{x}) + h(\mathbf{x})\}$$

model prior

- Let's take a closer look at these two proximal algorithms

FISTA

$$\mathbf{z}^k \leftarrow \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

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ADMM

$$\mathbf{z}^k \leftarrow \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k \leftarrow \text{prox}_{\gamma h}(\mathbf{z}^k + \mathbf{s}^{k-1})$$

$$\mathbf{s}^k \leftarrow \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

Plug and Play Prior (PnP) [Venkat'13]:

simply replace the proximal map with other denoisers \mathbf{D}_σ !

$$\text{prox}_{\gamma h} \Rightarrow \mathbf{D}_\sigma$$

where $\sigma \geq 0$ refers to denoising strength.

PnP: Incorporating a denoiser in the optimization

- Plug-and-Play (PnP) embraces off-the-shelf image denoisers

PnP-FISTA

$$\mathbf{z}^k = \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k)$$

$$\mathbf{s}^k = \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$

any off-the-shelf
image denoiser

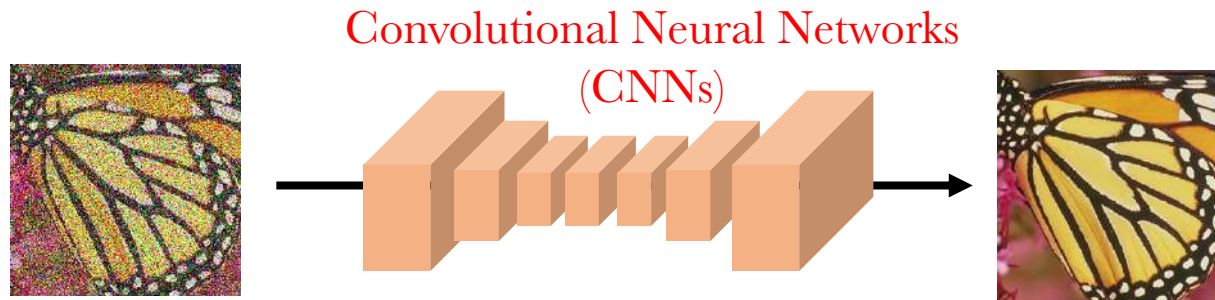
PnP-ADMM

$$\mathbf{z}^k = \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k + \mathbf{s}^{k-1})$$

$$\mathbf{s}^k = \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

Example: \mathbf{D}_σ could be a neural network



PnP: Incorporating a denoiser in the optimization

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$$\mathbf{z}^k = \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

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$$\mathbf{s}^k = \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$

$\sigma = ?$

PnP-ADMM

$$\mathbf{z}^k = \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k + \mathbf{s}^{k-1})$$

$$\mathbf{s}^k = \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

However, ...

- ❖ Many CNNs denoisers do not have a tunable parameter for the noise standard deviation!

PnP: Incorporating a denoiser in the optimization

- Plug-and-Play (PnP) embraces off-the-shelf image denoisers

PnP-FISTA

$$\mathbf{z}^k = \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k)$$

$$\mathbf{s}^k = \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$

$\sigma = ?$

PnP-ADMM

$$\mathbf{z}^k = \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k + \mathbf{s}^{k-1})$$

$$\mathbf{s}^k = \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

- Previous solution : denoiser selection
 - ⊛ Idea: Training multiple CNN instances and select the one that works best.
 - ⊛ Issues: Requires training multiple CNN instances and leads to suboptimal performance.

Proposed denoiser scaling technique

- Plug-and-Play (PnP) embraces off-the-shelf image denoisers

PnP-FISTA

$$\mathbf{z}^k = \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k)$$

$$\mathbf{s}^k = \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$

$\sigma = ?$

PnP-ADMM

$$\mathbf{z}^k = \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k + \mathbf{s}^{k-1})$$

$$\mathbf{s}^k = \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

- **Our proposal [Xu'20(1)] : denoiser scaling**

★ Introduce a tunable parameter μ to adjust the denoising strength of a pre-trained CNN.

Without scaling: $\hat{\mathbf{z}} = \mathbf{D}_\sigma(\mathbf{z})$

Denoiser scaling: $\hat{\mathbf{z}} = \mu^{-1} \mathbf{D}_\sigma(\mu \mathbf{z}), \quad \mu > 0$

Performance of denoiser scaling

- CNN trained on noise level $\sigma = 20$, applied on noise level $\sigma = 30$, difference $\Delta\sigma = 10$.

Noisy image:

z

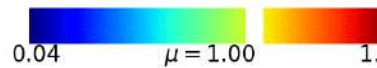
Corrupted (SNR = 9.58 dB)



Without scaling:

$$\hat{z} = D_{\sigma}(z)$$

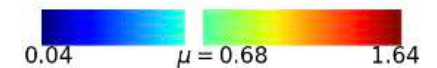
Unscaled (SNR = 17.93 dB)



With scaling:

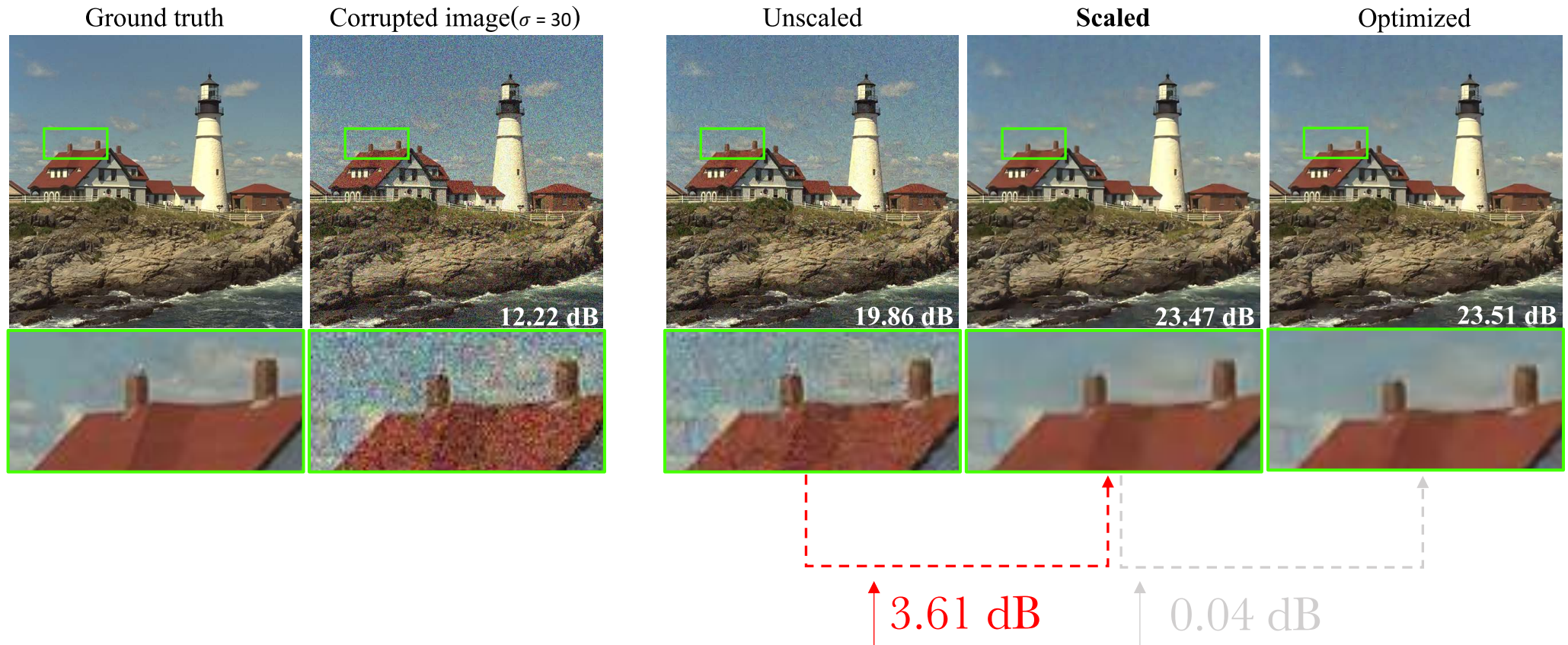
$$\hat{z} = \mu^{-1} D_{\sigma}(\mu z)$$

Scaled (SNR = 22.81 dB)



Performance of denoiser scaling

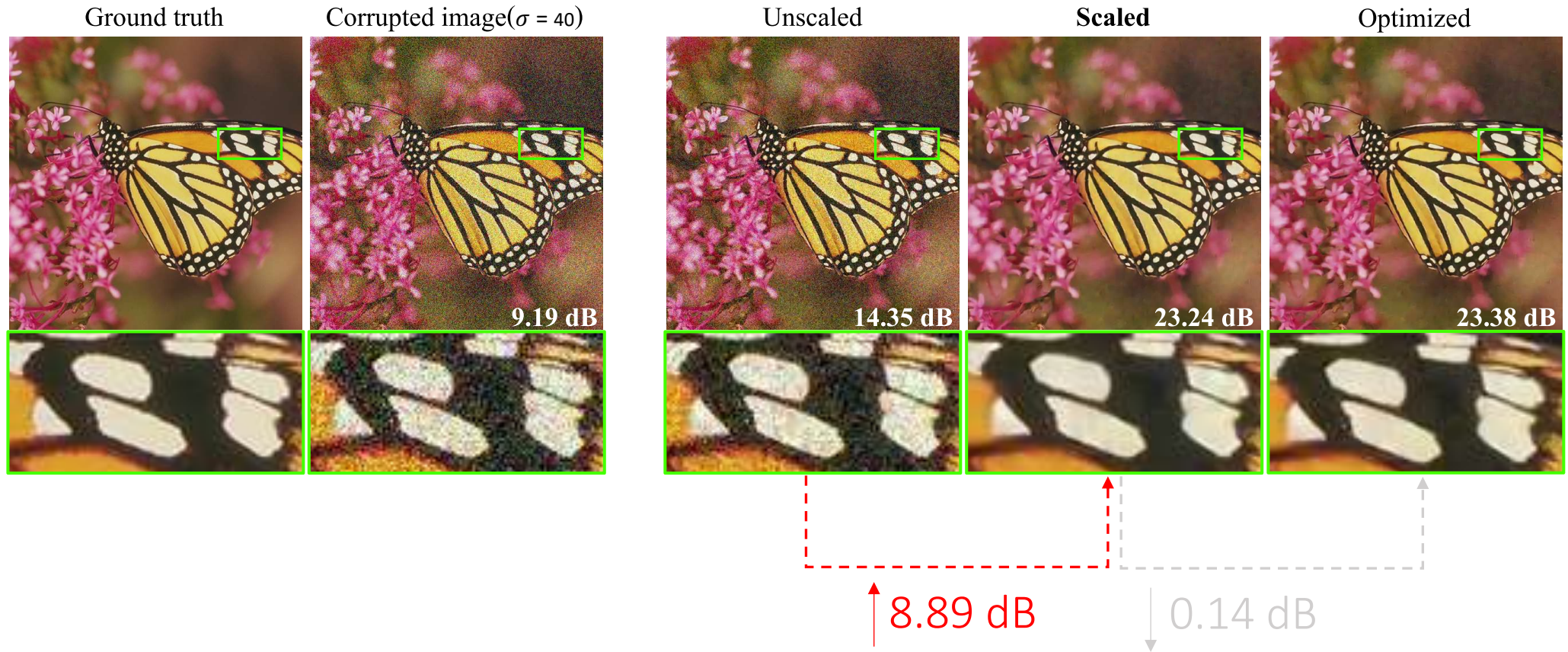
- CNN trained on noise level $\sigma = 20$, applied on noise level $\sigma = 30$, difference $\Delta\sigma = 10$.



*Number written to image is signal-to-noise ratio (SNR)

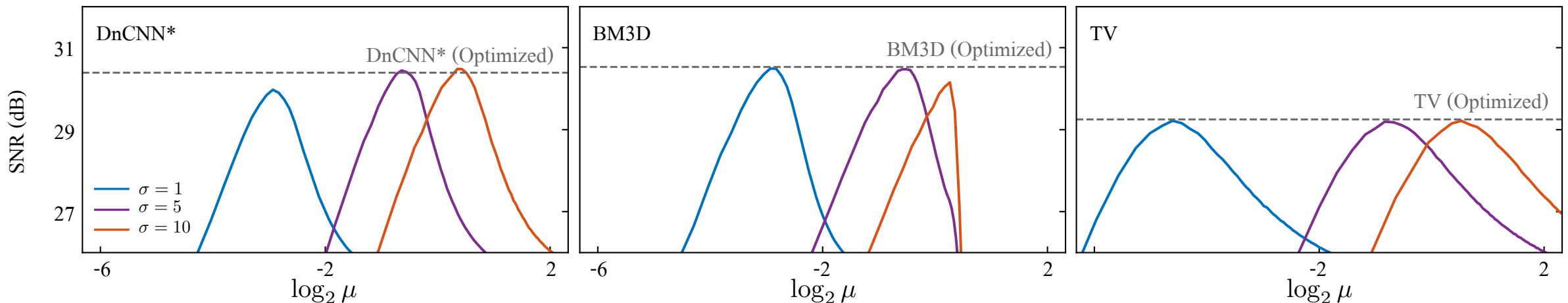
Performance of denoiser scaling

- CNN trained on noise level $\sigma = 20$, applied on noise level $\sigma = 40$, difference $\Delta\sigma = 20$.



Theoretical analysis of denoiser scaling

- **Denoiser scaling** is proved to have the following properties:
 - ★ When the denoiser is a minimum mean-squared error (MMSE) denoiser, adjusting μ is equivalent to scale the variance of AWGN by μ^{-2} in the MMSE estimation.
 - ★ When denoiser is a proximal map $\text{prox}_{\lambda h}(z) := \arg \min \{ \frac{1}{2} \|x - z\|_2^2 + \lambda h(x) \}$, where regularizer $h(\cdot)$ is 1-homogeneous with $h(\mu \cdot) = \mu h(\cdot)$, adjusting μ is equivalent to adjusting the weighting parameter λ of h .



PnP algorithms with denoiser scaling

- PnP algorithms with denoiser scaling

PnP-FISTA

$$\mathbf{z}^k = \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma(\mathbf{z}^k)$$

$$\mathbf{s}^k = \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$



Scaled PnP-FISTA

$$\mathbf{z}^k = \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mu^{-1} \mathbf{D}_\sigma(\mu \mathbf{z}^k)$$

$$\mathbf{s}^k = \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})$$

PnP-ADMM

$$\mathbf{z}^k = \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mathbf{D}_\sigma((\mathbf{z}^k + \mathbf{s}^{k-1}))$$

$$\mathbf{s}^k = \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$



Scaled PnP-ADMM

$$\mathbf{z}^k = \text{prox}_{\gamma g}(\mathbf{x}^{k-1} - \mathbf{s}^{k-1})$$

$$\mathbf{x}^k = \mu^{-1} \mathbf{D}_\sigma(\mu(\mathbf{z}^k + \mathbf{s}^{k-1}))$$

$$\mathbf{s}^k = \mathbf{s}^{k-1} + (\mathbf{z}^k - \mathbf{x}^k)$$

Inverse problem examples

- Image Super-resolution (SR) and Magnetic resonance imaging (MRI) problem

Low-resolution image

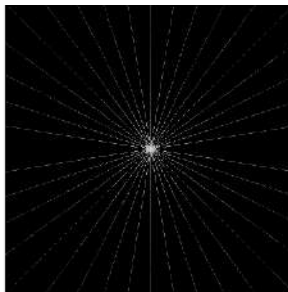


SR inverse problem

High-resolution image

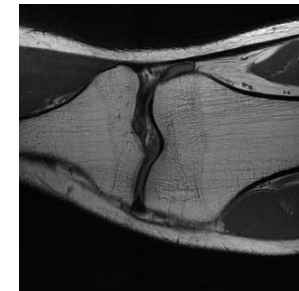


Under-sampled frequencies



MRI inverse problem

Clean image



Scaling performance in image SR problem

- Scaling technique can sharpen the blurry edges caused by the suboptimal denoiser.

Unscaled CNN



Scaled CNN (Ours)



Optimized

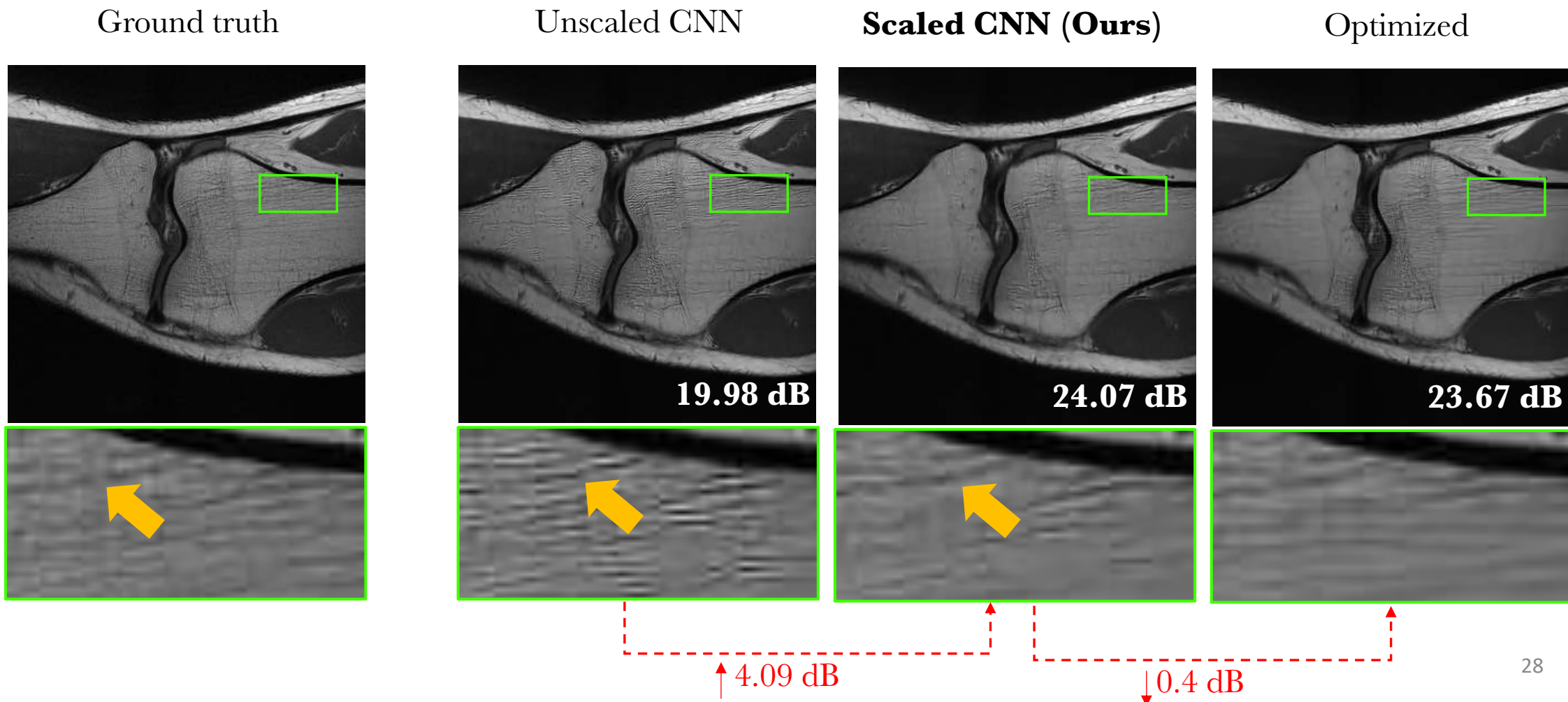


↑ 0.54 dB

↓ 0.04 dB

Scaling performance in MRI problem

- Scaling technique can alleviate the artifacts caused by the suboptimal denoiser.



Theoretical challenge of PnP scheme

Good!

- ❖ Denoiser scaling can effectively boost the performance of PnP algorithms and achieve the optimal results for different popular denoisers!

However,

- ❖ PnP algorithm lose the interpretation as an optimization problem for an arbitrary denoiser.

Theoretical challenge of PnP scheme

- PnP algorithm lose the interpretation as an optimization problem for an arbitrary denoiser.

FISTA

$$\begin{aligned}
 \mathbf{z}^k &\leftarrow \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1}) \\
 \mathbf{x}^k &\leftarrow \text{prox}_{\gamma h}(\mathbf{z}^k) \quad \text{proximal map} \\
 \mathbf{s}^k &\leftarrow \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})
 \end{aligned}$$

objective function

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \{g(\mathbf{x}) + \text{prior } h(\mathbf{x})\}$$

PnP-FISTA

$$\begin{aligned}
 \mathbf{z}^k &= \mathbf{s}^{k-1} - \gamma \nabla g(\mathbf{s}^{k-1}) \\
 \mathbf{x}^k &= D_{\sigma}(\mathbf{z}^k) \quad \text{any off-the-shelf denoiser} \\
 \mathbf{s}^k &= \mathbf{x}^k + ((q_{k-1} - 1)/q_k)(\mathbf{x}^k - \mathbf{x}^{k-1})
 \end{aligned}$$

objective function

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \{g(\mathbf{x}) + \text{prior } ?\}$$

For most off-the-shelf denoisers, it is impossible to write an explicit regularizer!

Provable Convergence of PnP with MMSE denoisers

- Can we build a relationship between some cost function f and some type of denoisers when running PnP?

$$f(\mathbf{x}) = g(\mathbf{x}) + ?$$

- Yes! Let's consider a MMSE denoiser:

$$\mathbf{D}_\sigma(\mathbf{y}) = \mathbb{E}[\mathbf{x} | \mathbf{y}] \quad \text{where} \quad \mathbf{y} = \mathbf{x} + \mathbf{e}, \quad \mathbf{e} \sim \mathcal{N}(0, \sigma^2 \mathbf{I}), \quad \mathbf{x} \sim p_{\mathbf{x}}.$$

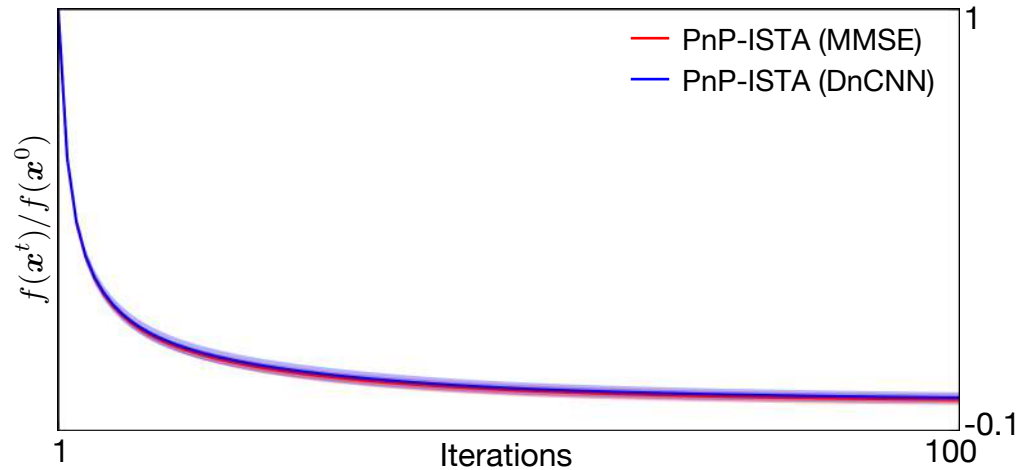
- Convergence of PnP-ISTA with MMSE denoisers [For details, see Xu' 20(2)]

- The iterates produced by PnP-ISTA with an MMSE denoiser converge to a stationary point of some global cost function.

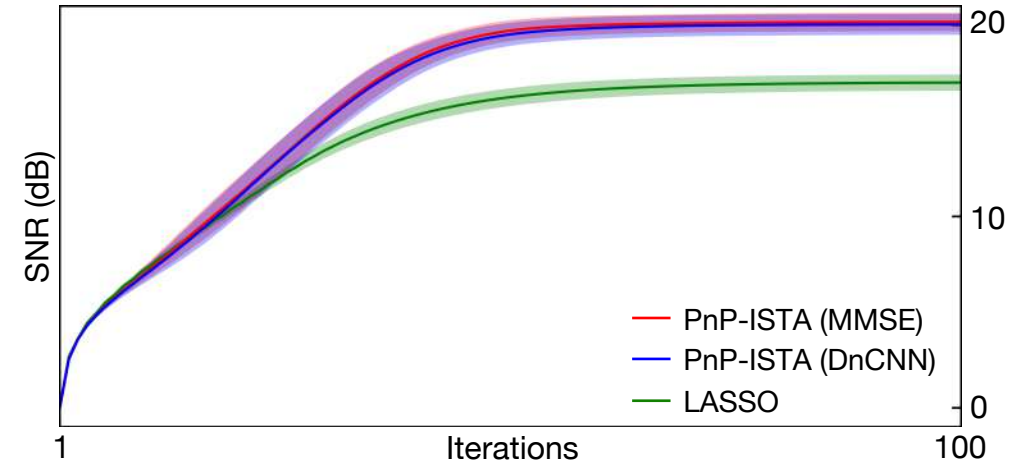
$$f(\mathbf{x}) = g(\mathbf{x}) + h(\mathbf{x}) \quad \text{with} \quad h(\mathbf{x}) := \begin{cases} -\frac{1}{2\gamma} \|\mathbf{x} - \mathbf{D}_\sigma^{-1}(\mathbf{x})\|^2 + \frac{\sigma^2}{\gamma} h_\sigma(\mathbf{D}_\sigma^{-1}(\mathbf{x})) & \text{for } \mathbf{x} \in \mathcal{X} \\ +\infty & \text{for } \mathbf{x} \notin \mathcal{X}, \end{cases}$$

Provable Convergence of PnP with MMSE denoisers

- Convergence and reconstruction performance of using MMSE denoiser and DnCNN for Bernoulli-Gaussian signals in compressive sensing



Convergence of PnP-ISTA for exact and approximate MMSE denoisers on loss function.



Convergence of PnP-ISTA for exact and approximate MMSE denoisers on SNR.

Summary of the talk

1. Introduction to Plug-and-Play Priors (PnP)

- Denoiser strength selecting challenge
- Optimization interpretation and convergence analysis challenge

2. Denoiser scaling technique [Xu'20(1)]

- We proposed a denoiser scaling technique that can help with the denoising strength tuning especially for CNN type of denoisers.
- We showed that denoiser scaling can effectively boost the performance of PnP algorithms and achieve the optimal results.

3. Optimization interpretation and convergence analysis of PnP with MMSE denoisers [Xu'20(2)]

- We show that the iterates produced by PnP-ISTA with an MMSE denoiser converge to a stationary point of some global cost function.

Additional key citations

- [Chan'16]: Chan et.al. Plug-and-play ADMM for image restoration: Fixed-point convergence and applications, *IEEE TCI*, 2016
- [Venkat'13]: Venkat et.al. Plug-and-play priors for model based reconstruction, *IEEE GlobalSIP*, 2013.
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- [Sun'19]: Sun et.al. Block coordinate regularization by denoising, *NeurIPS*, 2019.
- [Xia'19]: Xia et.al. Training Image Estimators without Image Ground-Truth, *NeurIPS*, 2019
- [Lee'18]: Lee et al. Deep residual learning for accelerated MRI using magnitude and phase networks, *Trans. Bio. Eng.* 2018
- [Ryu'19]: Ryu et al. Plug-and-play methods provably converge with properly trained denoisers, *ICML* 2019
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- [Aggarwal'18]: Aggarwal et al. MoDL: Model-based deep learning architecture for inverse problems, *IEEE TMI*, 2018
- [Zhang'18]: Zhang et al. ISTA-Net: Interpretable optimization-inspired deep network for image compressive sensing, *ICCV*, 2018
- [Yang'16]: Yang et al. Deep ADMM-Net for compressive sensing MRI *NeurIPS*, 2016.
- [Liu'19]: Liu et al. Infusing Learned Priors into Model-Based Multispectral Imaging, *arXiv preprint arXiv:1909.09313* (2019).

Thanks!