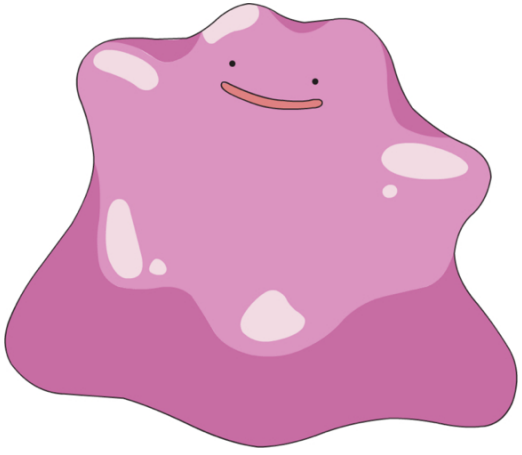
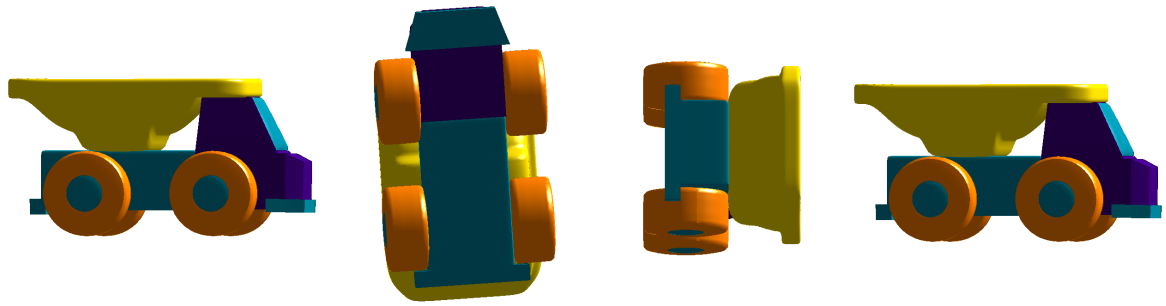
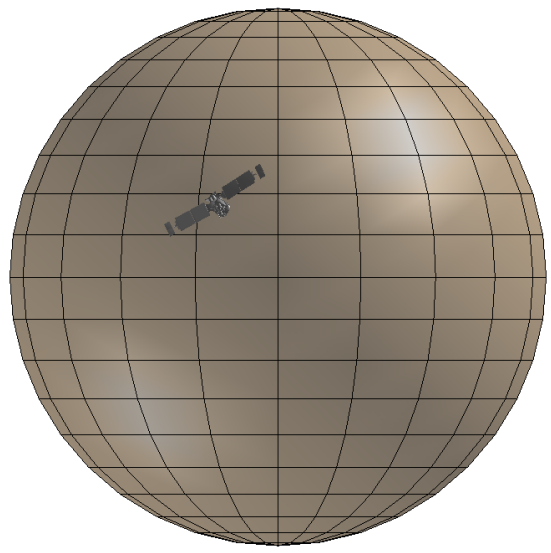
About interpolation on manifolds...



About interpolation on manifolds...



About interpolation on manifolds...

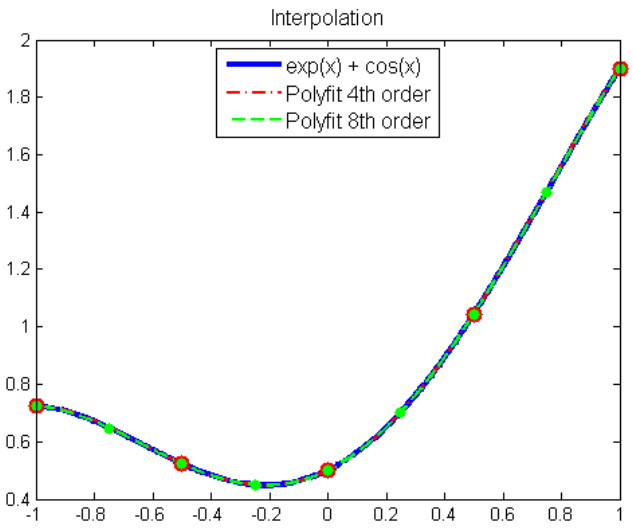


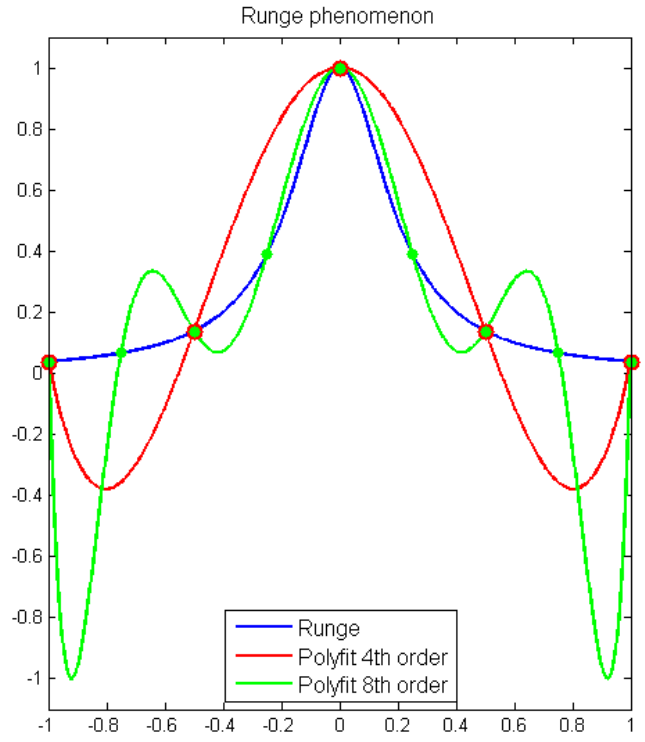
How to interpolate points on curved spaces ?

Light **fast** **general** **good looking** **interpolation**

About interpolation on manifolds...

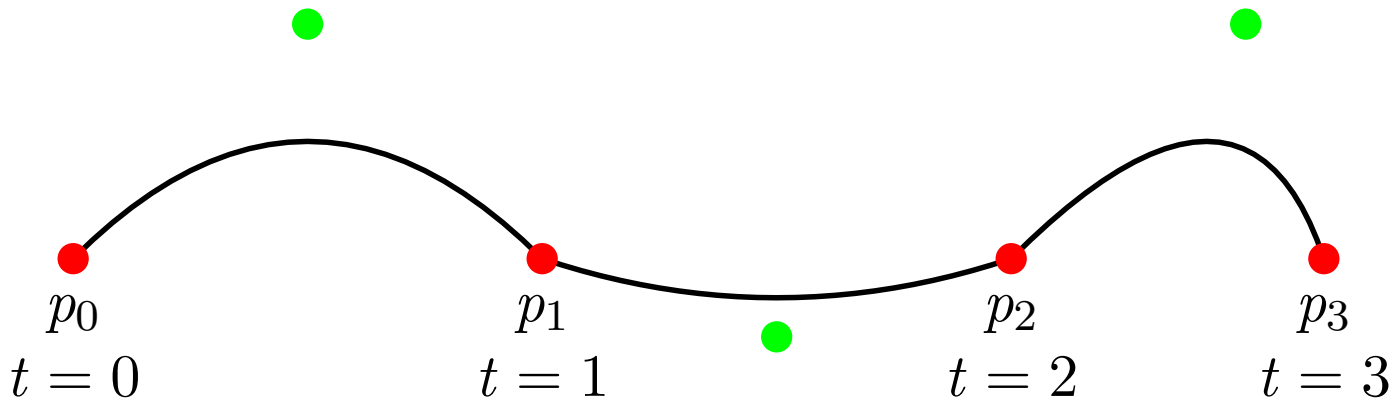
The Euclidean space is a manifold too but with many solutions for interpolation.





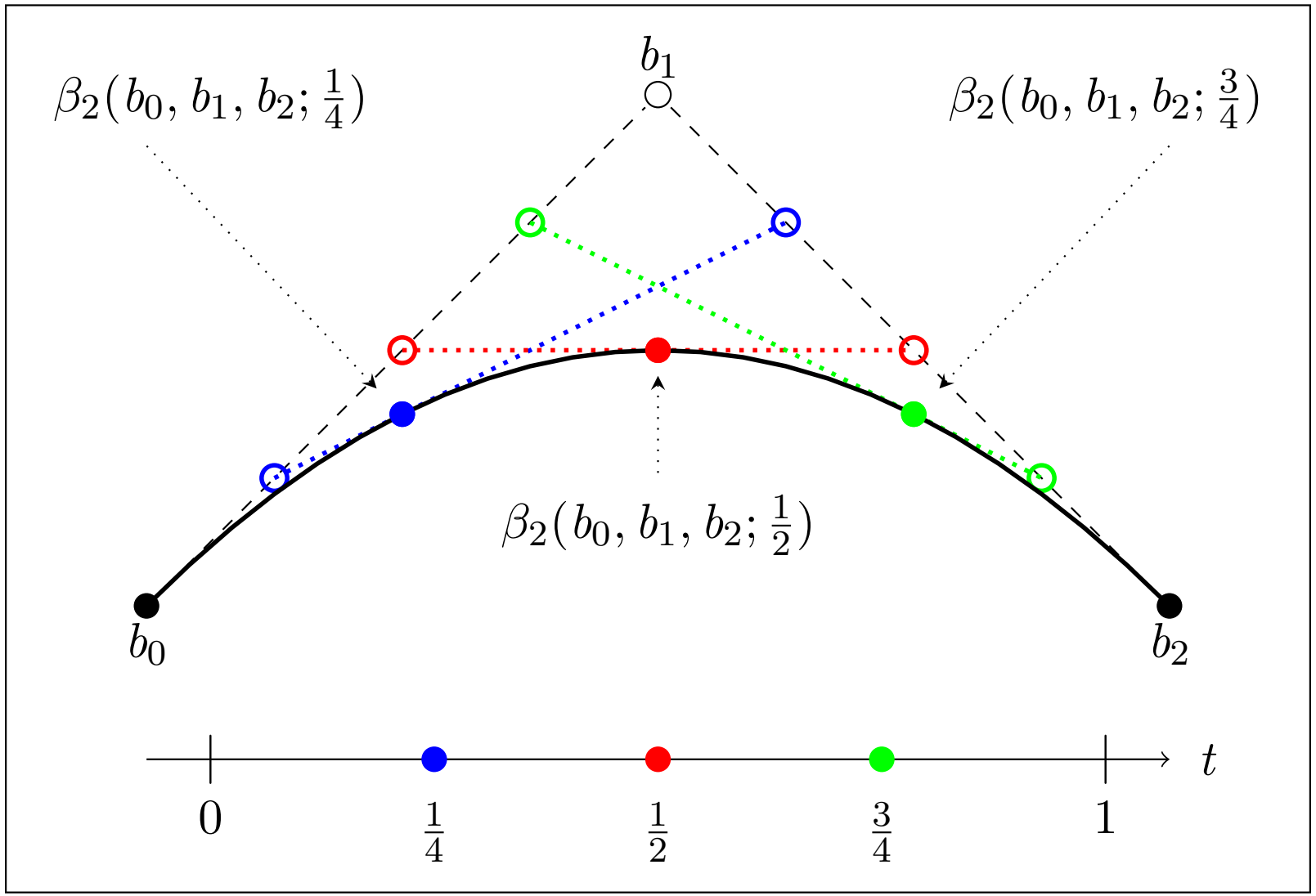
How to interpolate ?

Each segment between two consecutive points is a Bézier function.



Light **fast** **general** **good looking** **interpolation**

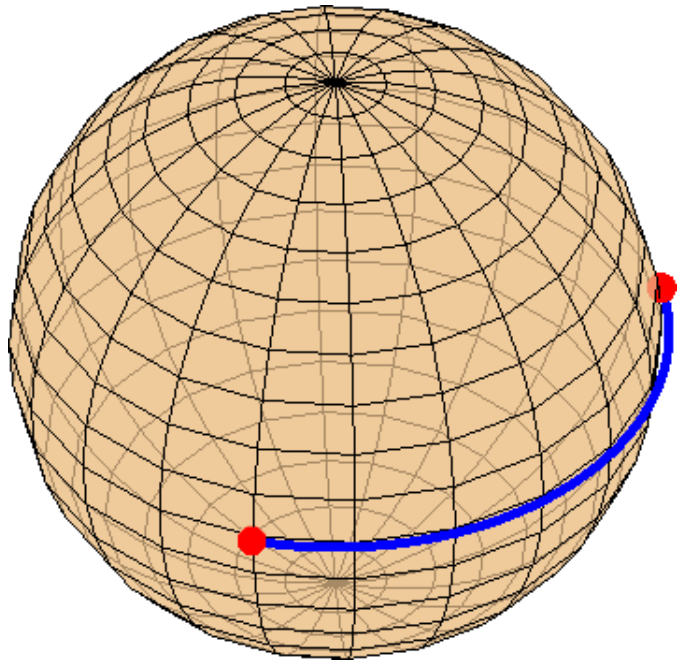
Reconstruction : the De Casteljau algorithm

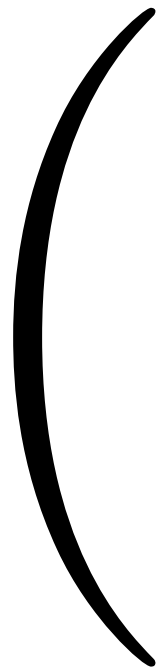


Light **fast** general good looking **interpolation**

How to generalize Bézier curves to manifolds ?

The straight line is a geodesic

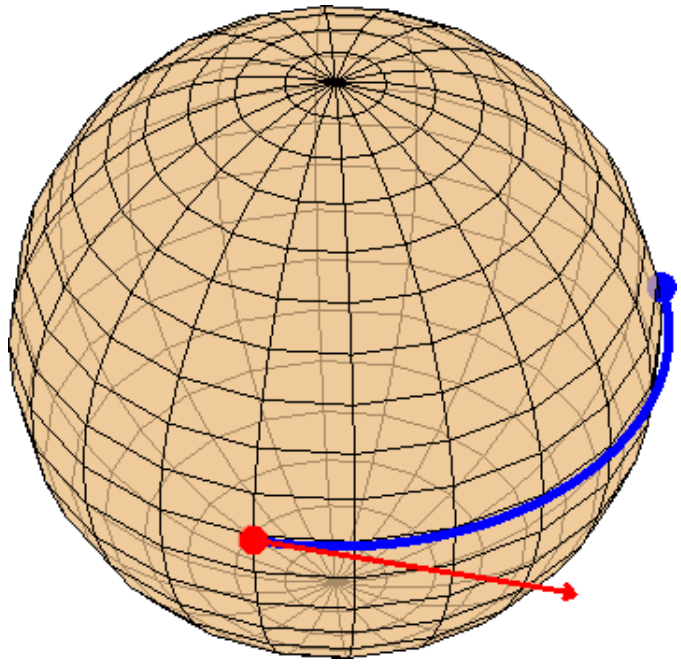




How to generalize Bézier curves to manifolds ?

The exponential map to construct the geodesic

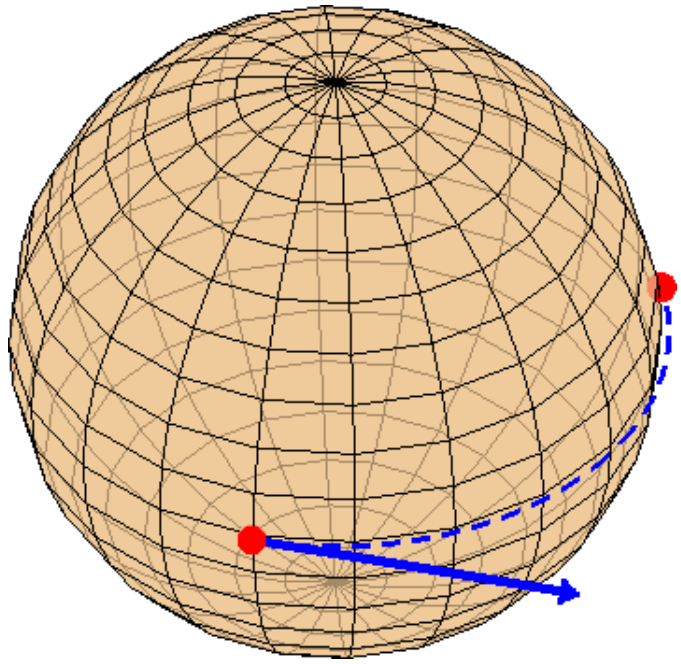
$$\gamma(t) = \mathrm{Exp}_x(t\xi_x)$$

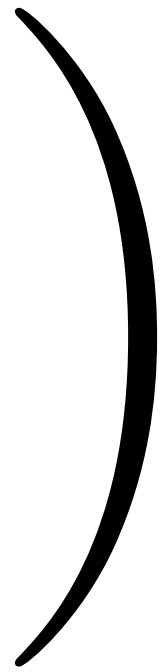


How to generalize Bézier curves to manifolds ?

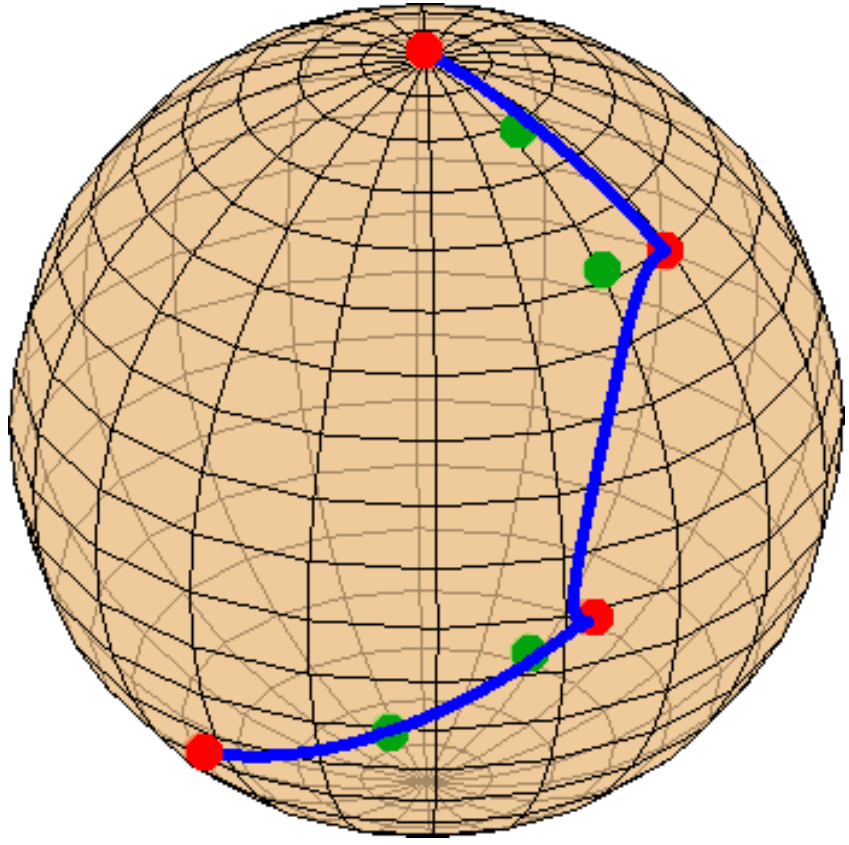
The logarithmic map to determine the starting velocity

$$\mathrm{Log}_x(y) = \xi_x$$





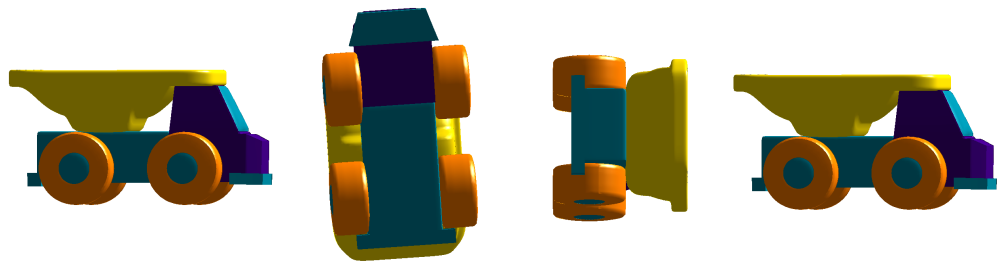
Piecewise interpolation on the sphere



Light **fast** **general** **good looking** **interpolation**

Interpolation on Riemannian manifolds with a $\mathcal{C}^1$ piecewize-Bézier path

Pierre-Yves Gousenbourger



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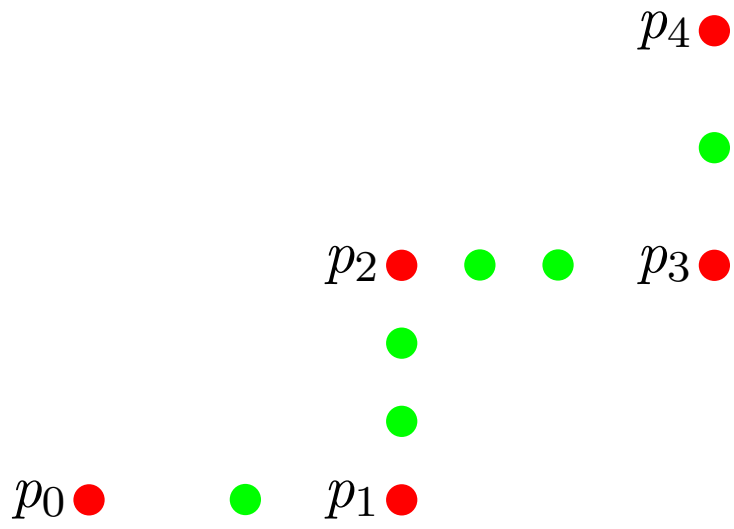
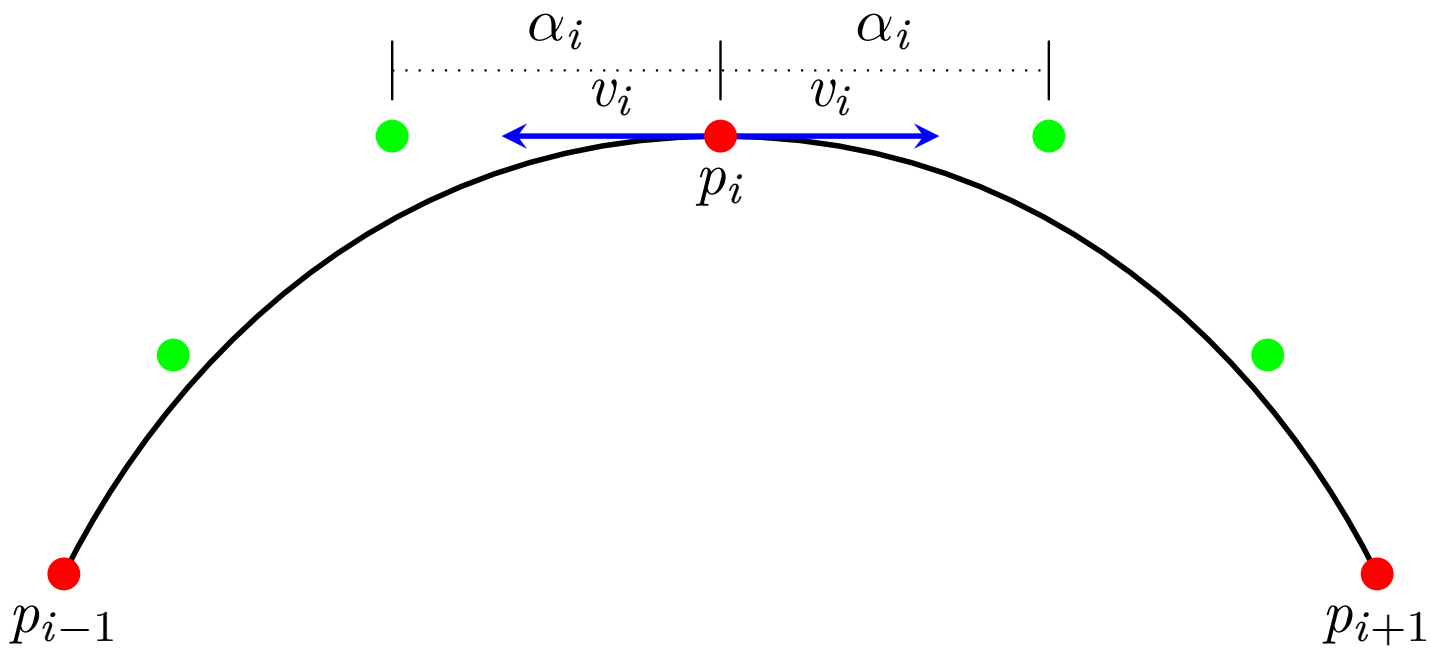
Good-looking curve on the Euclidean space



Find the optimal position of control points

$\mathcal{C}^1$-piecewise Bézier interpolation



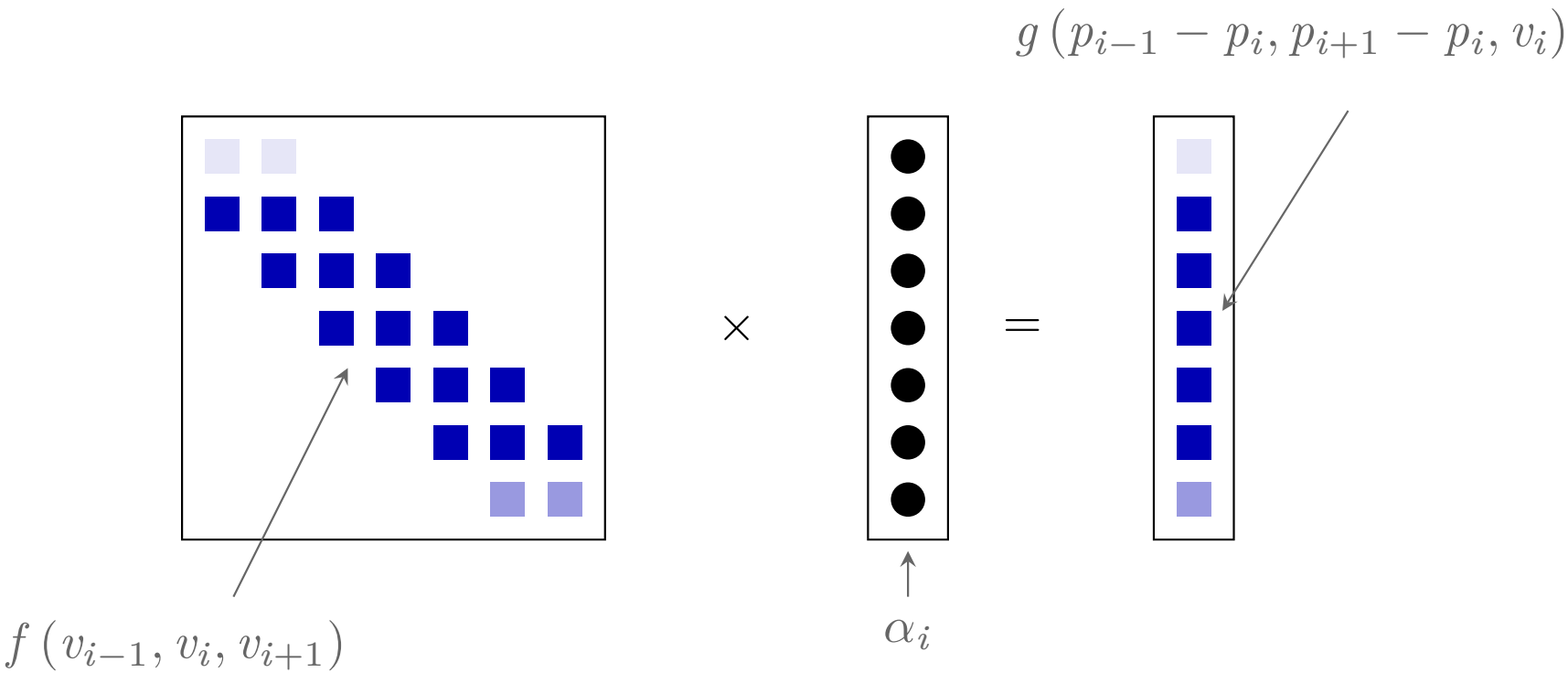
$$b_i^L = \mathrm{Exp}_{p_i}(-\alpha_\mathbf{i} v_i)$$

$$b_i^R = \mathrm{Exp}_{p_i}(\quad \alpha_\mathbf{i} v_i)$$

Optimal $\mathcal{C}^1$-piecewise Bézier interpolation

Minimization of the mean square acceleration of the path

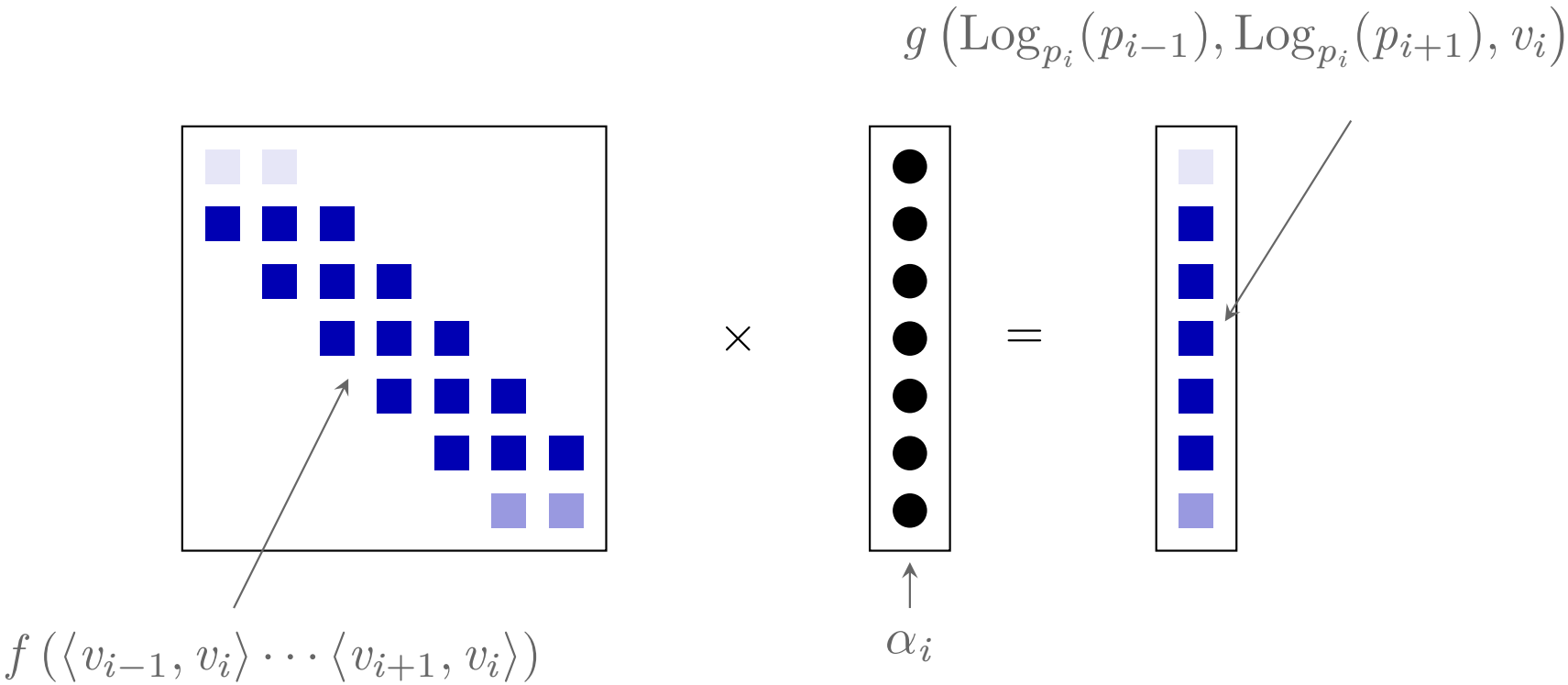
$$\min_{\alpha_i} \|\ddot{\beta}_2^0(b_j;\mathbf{t})\|^2 + \sum_{i=1}^{n-1} \|\ddot{\beta}_3^i(b_j;\mathbf{t})\|^2 + \|\ddot{\beta}_2^n(b_j;\mathbf{t})\|^2$$



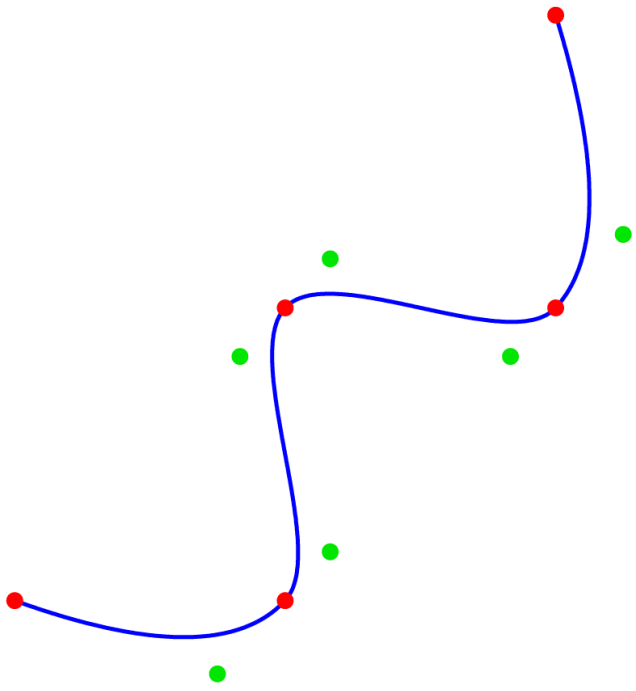
Optimal $\mathcal{C}^1$-piecewise Bézier interpolation

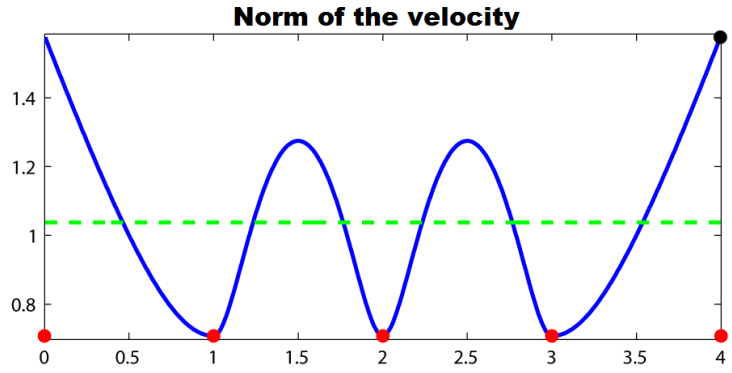
Minimization of the mean square acceleration of the path

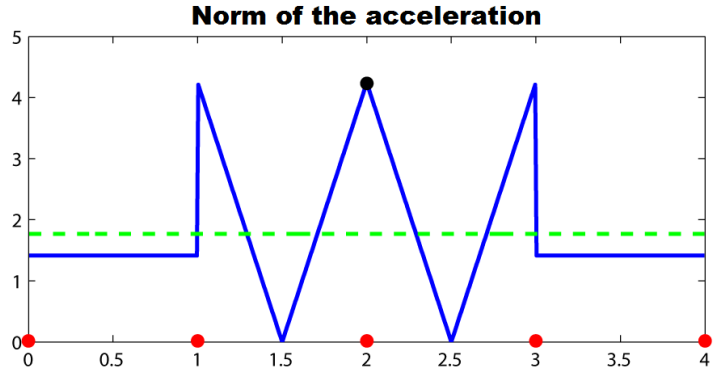
$$\min_{\alpha_i} \|\ddot{\beta}_2^0(b_j;\mathbf{t})\|^2 + \sum_{i=1}^{n-1} \|\ddot{\beta}_3^i(b_j;\mathbf{t})\|^2 + \|\ddot{\beta}_2^n(b_j;\mathbf{t})\|^2$$



A result on $\mathbb{R}^2$

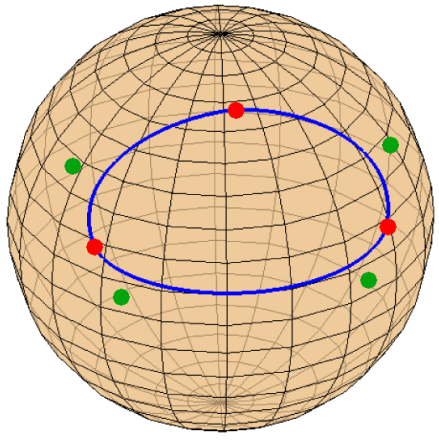


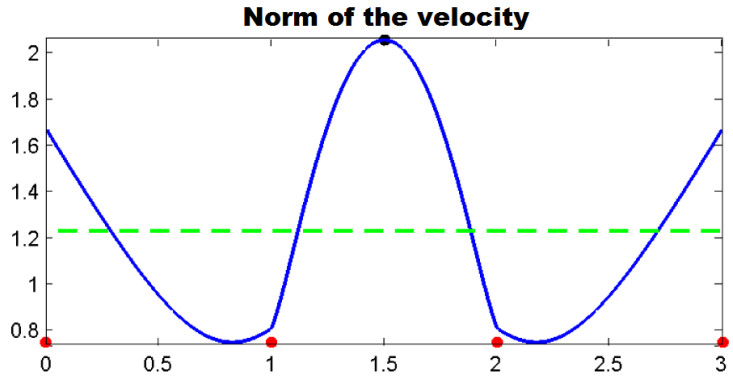


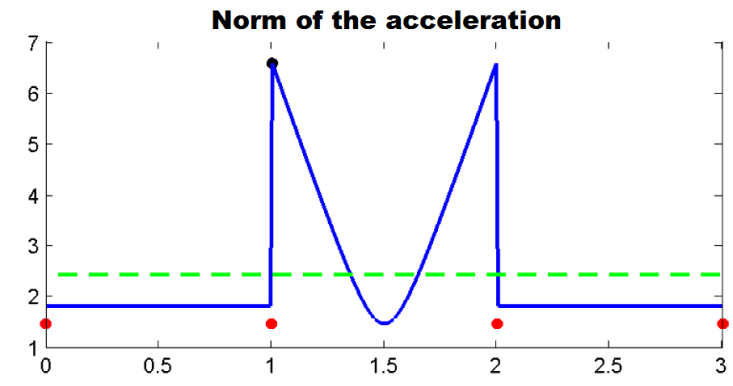


Light fast general good looking interpolation

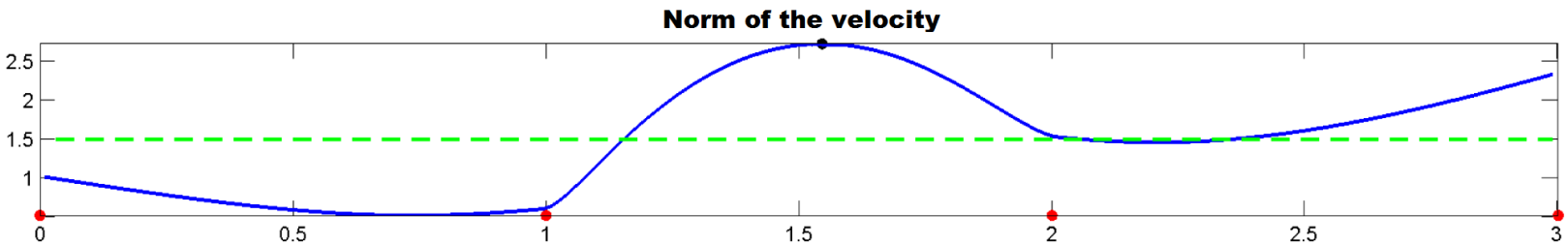
Generalization to manifolds : the sphere $\mathbb{S}^2$

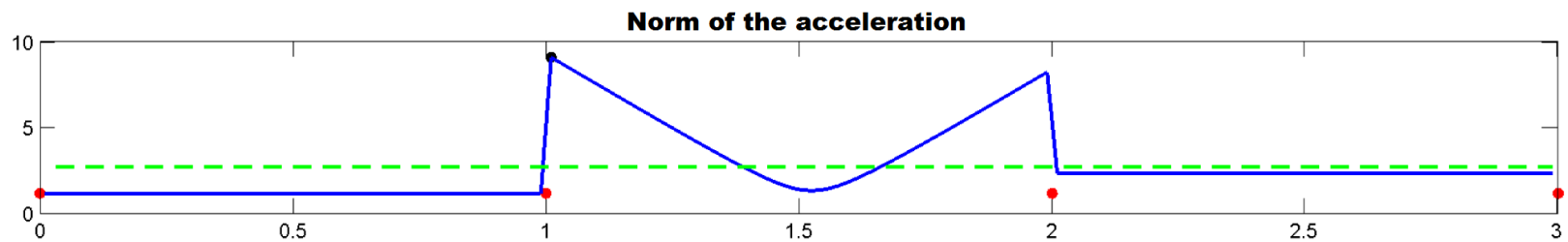




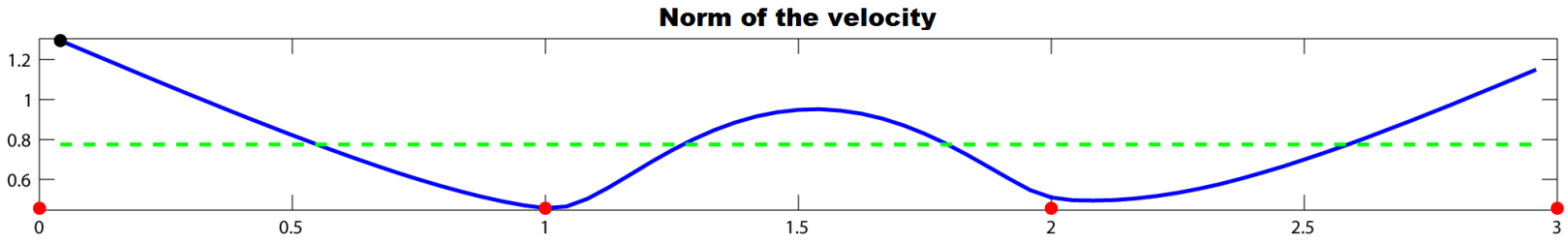


Generalization to manifolds : the special orthogonal group $SO(3)$





Generalization to manifolds : morphing of shapes



Conclusions

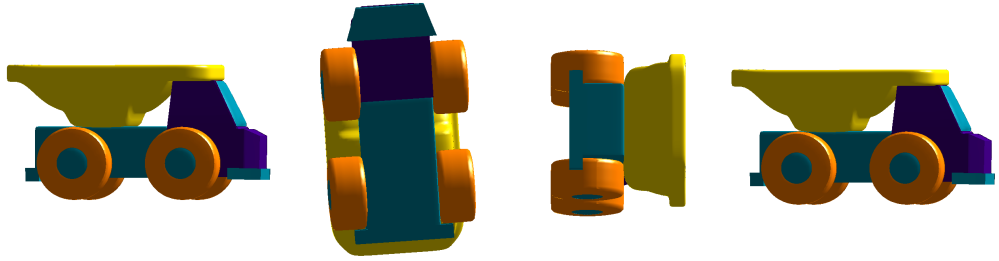
Light fast general good looking interpolation

Choice of the velocities v_i ?
Application to manifolds of high dimension ?
Other methods, different from Bézier ?

Any questions ?

Interpolation on Riemannian manifolds with a $\mathcal{C}^1$ piecewize-Bézier path

Pierre-Yves Gousenbourger



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