The lensless endoscope:

a playground for acquisition schemes

based on compressed sensing principles

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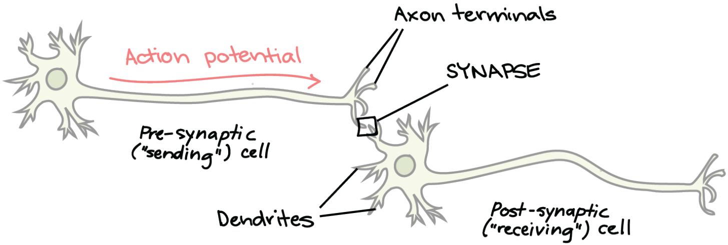
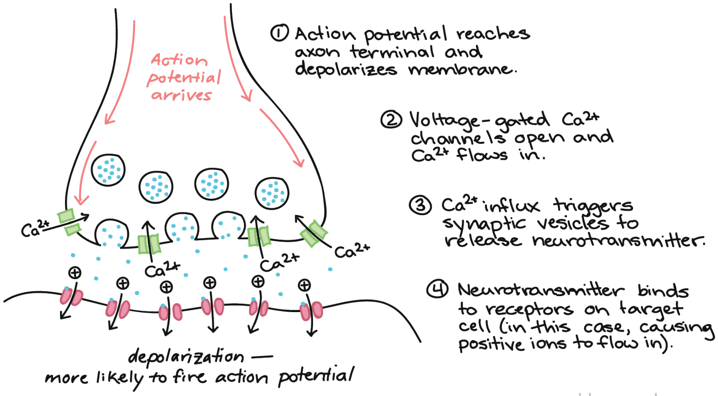
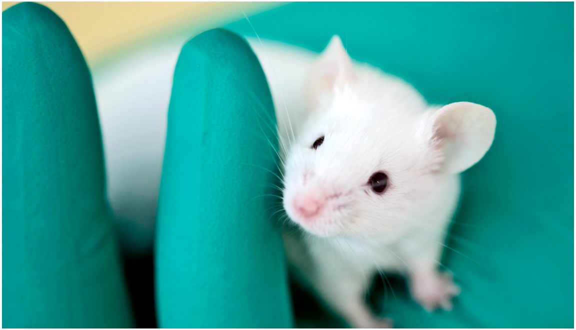
Collaborators: S. Sivankutty, C. Scotte, J. A. Lee, H. Rigneault, and L. Jacques

ISPSeminar, 5th of June


Axon terminals
Action potential
SYNAPSE
Pre-synaptic
("sending") cell
Dendrites
Post-synaptic
("receiving") cell


Action potential arrives
Ca2+
Ca2+
Ca2+
Ca2+
depolarization — more likely to fire action potential
① Action potential reaches axon terminal and depolarizes membrane.
② Voltage-gated Ca2+ channels open and Ca2+ flows in.
③ Ca2+ influx triggers synaptic vesicles to release neurotransmitter.
④ Neurotransmitter binds to receptors on target cell (in this case, causing positive ions to flow in).



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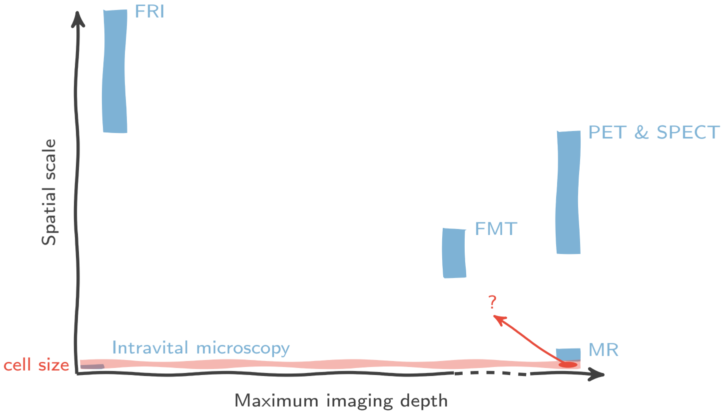
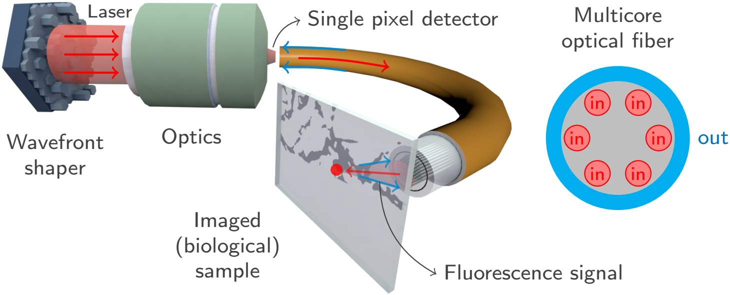
Exploring cells and quantifying cellular processes *in vivo*

Current performances of **molecular imaging** devices for small animals



M. Rudin and R. Weissleder, "Molecular imaging in drug discovery and development," *Nature Reviews Drug Discovery*, 2003

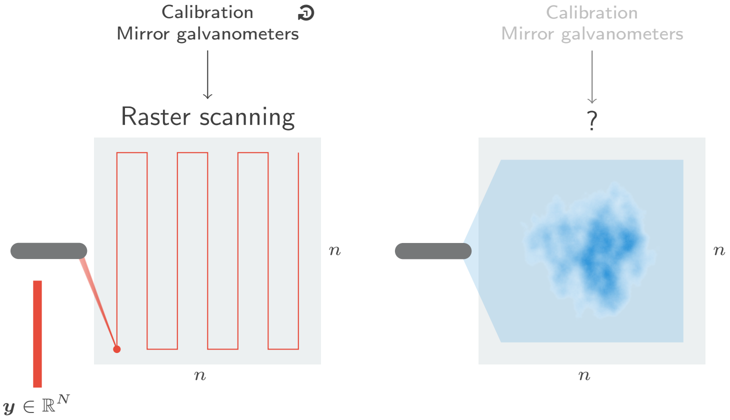
The lensless endoscope: an utltrathin device to image cells



E. R. Andresen, S. Sivankutty, V. Tsvirkun, *et al.*, "Ultrathin endoscopes based on multicore fibers and adaptive optics: status and perspectives," *Journal of Biomedical Optics*, 2016

S. Sivankutty, V. Tsvirkun, O. Vanvincq, *et al.*, "Nonlinear imaging through a fermat's golden spiral multicore fiber," *Optics letters*, 2018

Relaxing some constraints could accelerate the acquisition



$M = N = n \times n$

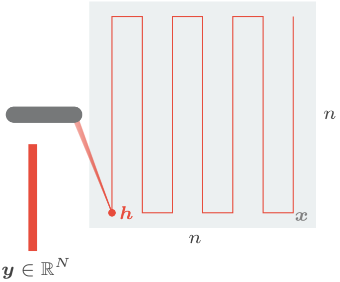
$M < N$?

The lensless endoscope:
a playground for acquisition schemes based on compressed sensing principles

Compressive sampling uses unstructured illumination patterns

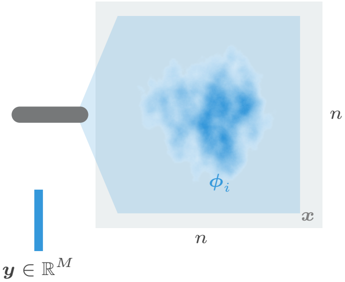
Calibration
Mirror galvanometers

Raster scanning



$$\boldsymbol{y} = \boldsymbol{h} \otimes x + \boldsymbol{n}$$

Compressive sampling



$$\boldsymbol{y} = \boldsymbol{\Phi x} + \boldsymbol{n}$$

$$\boldsymbol{\Phi} \in \mathbb{R}^{M \times N}$$

The inverse problem is expressed as a minimization

Compressive sampling

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{n}$$

Estimation problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}} \|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{z}\|_2^2$$

Ill-posed problems!

Adding priors reduces the set of feasible solutions for $\hat{\boldsymbol{x}}$

Compressive sampling

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{n}$$

Estimation problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}} \|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{z}\|_2^2 + \rho\,\boldsymbol{\Phi}(\boldsymbol{z})$$

Add priors on $\hat{\boldsymbol{x}}$...

We assume that the wavelet representation of $\hat{\boldsymbol{x}}$, $(\boldsymbol{\Psi}^T\hat{\boldsymbol{x}})$, is sparse.

Adding priors reduces the set of feasible solutions for $\hat{\boldsymbol{x}}$

Compressive sampling

$$\boldsymbol{y} = \boldsymbol{\Phi}x + \boldsymbol{n}$$

Estimation problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}} \|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{z}\|_2^2$$

$$+ \rho\, \boldsymbol{\Phi}(\boldsymbol{z})$$

Add priors on $\hat{\boldsymbol{x}}$...

$\Leftrightarrow$ $\|\boldsymbol{\Psi}^T\hat{\boldsymbol{x}}\|_1$ *is small.* r-Redundant Discrete Wavelet Transform

$\Leftrightarrow$ $\sum_{i=1}^{r} \|\boldsymbol{\psi}^T\boldsymbol{S}_i\hat{\boldsymbol{x}}\|_1$ *is small.* $\boldsymbol{\Psi}^T = \begin{bmatrix}\psi_1 & \psi_2 & \dots & \psi_r\end{bmatrix}^T,\ \boldsymbol{\psi}_i^T = \boldsymbol{\psi}^T\boldsymbol{S}_i$

Adding priors reduces the set of feasible solutions for $\hat{\boldsymbol{x}}$

Compressive sampling

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{n}$$

Estimation problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}} \|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{z}\|_2^2 + \rho \sum_{i=1}^{r} \|\boldsymbol{\psi}^T \boldsymbol{S}_i \boldsymbol{z}\|_1$$

Let's solve these minimizations!

Generalized Forward-Backward algorithm

$$\hat{\boldsymbol{x}} = \text{arg min}_{\boldsymbol{z}} \underbrace{\|\boldsymbol{y} - \boldsymbol{\Phi z})\|_2^2}_{f,\ \text{differentiable}} + \underbrace{\rho \sum_{i=1}^{r} \|\boldsymbol{\psi}^T \boldsymbol{S}_i \boldsymbol{z}\|_1}_{g=\sum_i g_i}$$

Estimation

$\boldsymbol{r} = \mathbf{1}, \gamma = 1.8/L, L$ is the Lipschitz constant of ∇f

$$\hat{\boldsymbol{x}}^k \leftarrow \underbrace{\text{prox}_{\gamma g} \underbrace{\left[\hat{\boldsymbol{x}}^{k-1} - \gamma \nabla f(\hat{\boldsymbol{x}}^{k-1})\right]}_{\text{minimize } f}}_{\text{minimize } g}$$

H. Raguet, J. Fadili, and G. Peyré, "Generalized forward-backward splitting," *SIAM Journal on Imaging Sciences*, 2013

Generalized Forward-Backward algorithm

$$\hat{\boldsymbol{x}} = \operatorname{arg\,min}_{\boldsymbol{z}} \underbrace{\|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{z})\|_2^2}_{f,\ \text{differentiable}} + \underbrace{\rho \sum_{i=1}^{r} \|\boldsymbol{\psi}^T \boldsymbol{S}_i \boldsymbol{z}\|_1}_{g = \sum_i g_i}$$

Estimation

$\boldsymbol{r} > \mathbf{1}, \gamma = 1.8/L, L$ is the Lipschitz constant of ∇f

for $\boldsymbol{i} = \mathbf{1}$ **to** $\boldsymbol{r}$

$$\boldsymbol{s}_i^k \leftarrow \boldsymbol{s}_i^{k-1} + \text{prox}_{\gamma g_i / w_i}\left[2\hat{\boldsymbol{x}}^{k-1} - \boldsymbol{s}_i^{k-1} - \gamma / w_i \nabla f(\hat{\boldsymbol{x}}^{k-1})\right] - \hat{\boldsymbol{x}}^k$$

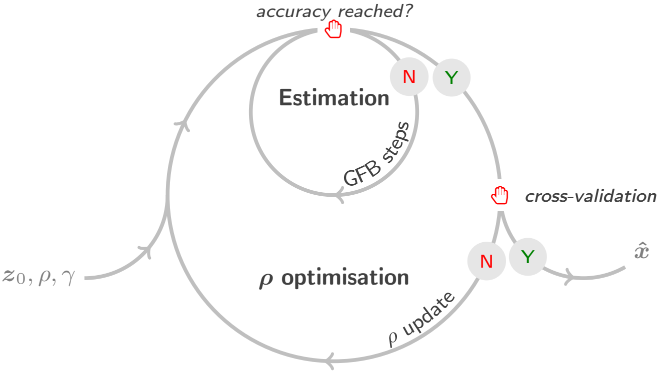
end

$$\hat{\boldsymbol{x}}^k = \sum_i w_i \boldsymbol{s}_i^k$$

H. Raguet, J. Fadili, and G. Peyré, "Generalized forward-backward splitting," *SIAM Journal on Imaging Sciences*, 2013

Generalized Forward-Backward algorithm

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}} \underbrace{\|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{z})\|_2^2}_{f,\ \text{differentiable}} + \underbrace{\rho \sum_{i=1}^{r} \|\boldsymbol{\psi}^T \boldsymbol{S}_i \boldsymbol{z}\|_1}_{g=\sum_i g_i}$$



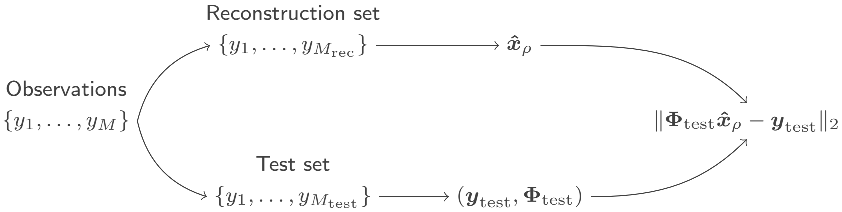
H. Raguet, J. Fadili, and G. Peyré, "Generalized forward-backward splitting," *SIAM Journal on Imaging Sciences*, 2013

Choice of ρ based on cross-validation

Aim: choosing ρ such that $\hat{\boldsymbol{x}}_\rho \approx \boldsymbol{x}$, *i.e.*, $\|\hat{\boldsymbol{x}}_\rho - \boldsymbol{x}\|_2$ is minimal.

Idea: if $\boldsymbol{\Phi}$ has "nice" properties,

$$\|\hat{\boldsymbol{x}}_\rho - \boldsymbol{x}\|_2 \leq C\|\boldsymbol{\Phi}\hat{\boldsymbol{x}}_\rho - \boldsymbol{\Phi}\boldsymbol{x}\|_2 \leq C(\underbrace{\|\boldsymbol{\Phi}\hat{\boldsymbol{x}}_\rho - \boldsymbol{y}\|_2}_{\text{computable!}} + \|\boldsymbol{n}\|_2), \quad C > 1.$$

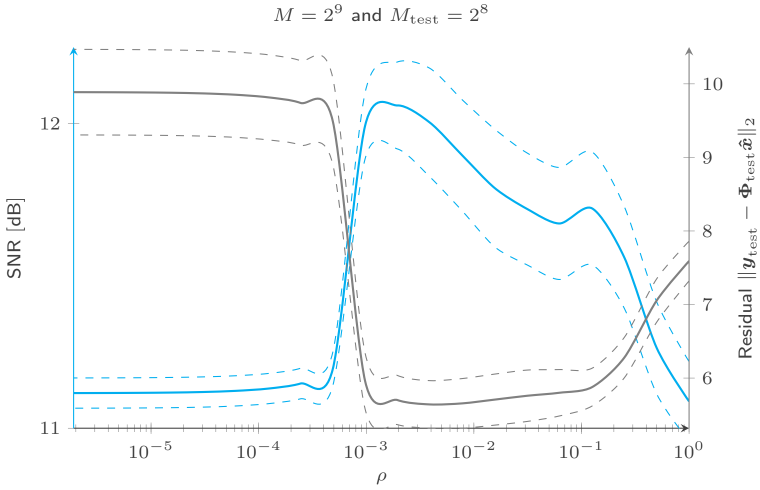


P. Boufounos, M. F. Duarte, and R. G. Baraniuk, "Sparse signal reconstruction from noisy compressive measurements using cross validation," in *2007 IEEE/SP 14th Workshop on Statistical Signal Processing*, Madison, WI, USA, 2007, ISBN: 0001406108

R. Ward, "Compressed sensing with cross validation," *IEEE Transactions on Information Theory*, 2009

Choice of ρ based on cross-validation

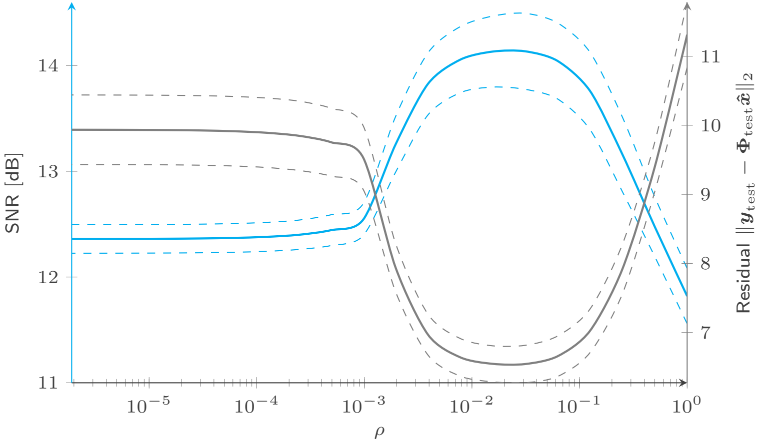
Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · 1 pattern · 20 trials · DWT



Choice of ρ based on cross-validation

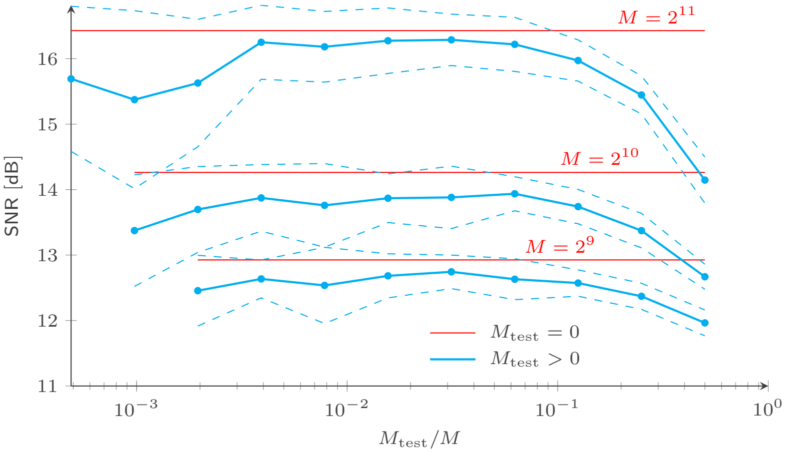
Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · 1 pattern · 20 trials · DWT

$M = 2^{11}$ and $M_{\text{test}} = 2^{8}$



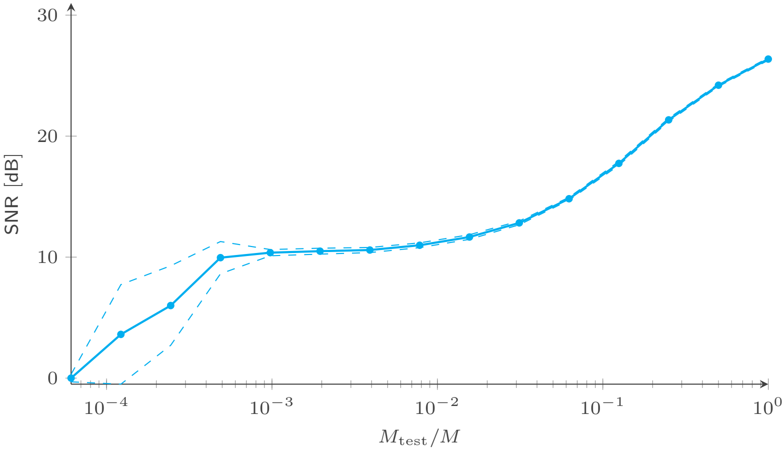
Choice of ρ based on cross-validation

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · 1 pattern · 20 trials · DWT



Synthetic experiment in CS framework

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · $M_{\text{test}} = 2^8$ · 5 trials · DWT



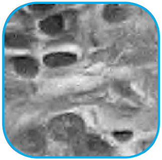
Synthetic experiment in CS framework

Synthetic data $\cdot$ $N = 128 \times 128$ $\cdot$ BSNR 40 dB $\cdot$ $M_{\text{test}} = 2^8$ $\cdot$ 5 trials $\cdot$ DWT

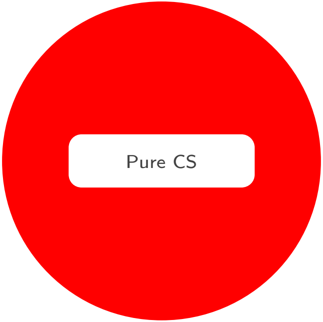
Raster scanning
$(M = N)$

CS: $M = 2^{10}$
6.25%

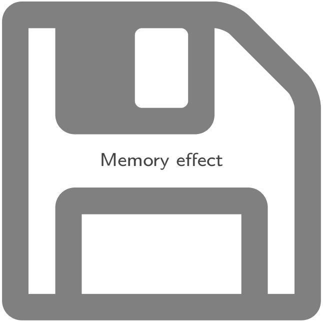
CS: $M = 2^{11}$
12.5%



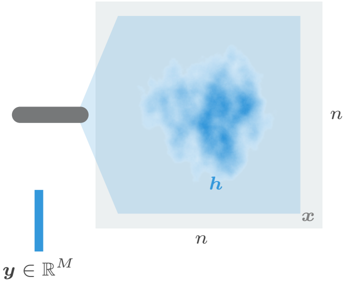
CS: $M = 2^{12}$
25%



Slow acquisition · Slow reconstruction · High memory

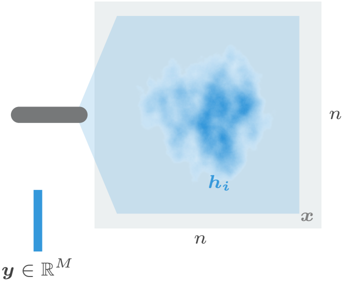


Fast acquisition? · Fast transformations? · Low memory?



$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{n}, \qquad \boldsymbol{\Phi} = \boldsymbol{R}_\Omega \boldsymbol{H}$$

$$\boldsymbol{R}_\Omega \in \mathbb{R}^{M \times N} \quad \Omega \subset \{1, \ldots, N\}$$
$$\boldsymbol{\mathcal{M}}_{\Omega_i} \in \mathbb{R}^{N \times N} \quad \cup_i \, \Omega_i = \Omega$$



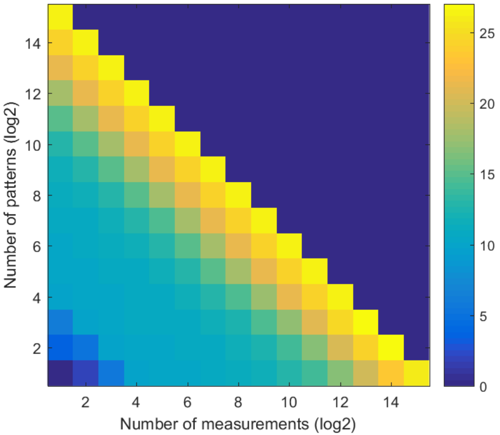
$$\boldsymbol{y} = \boldsymbol{\Phi} \boldsymbol{x} + \boldsymbol{n}, \qquad \boldsymbol{\Phi} = R_\Omega \sum_{i=1}^{P} \boldsymbol{\mathcal{M}}_{\Omega_i} \boldsymbol{H}_i$$

$$\boldsymbol{R}_\Omega \in \mathbb{R}^{M \times N} \quad \Omega \subset \{1, \ldots, N\}$$

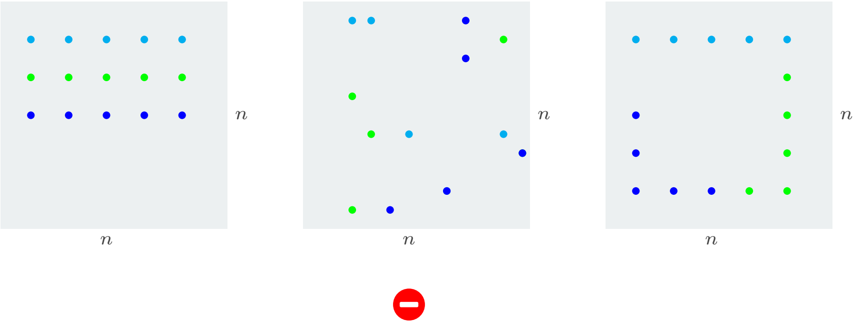
$$\boldsymbol{\mathcal{M}}_{\Omega_i} \in \mathbb{R}^{N \times N} \quad \cup_i \, \Omega_i = \Omega$$

How does M and P influence the quality of $\hat{x}$?

Synthetic data · $N = 128 \times 128$ · BSNR 40 dB · $M_{\text{test}} = 0$ · 20 trials · DWT



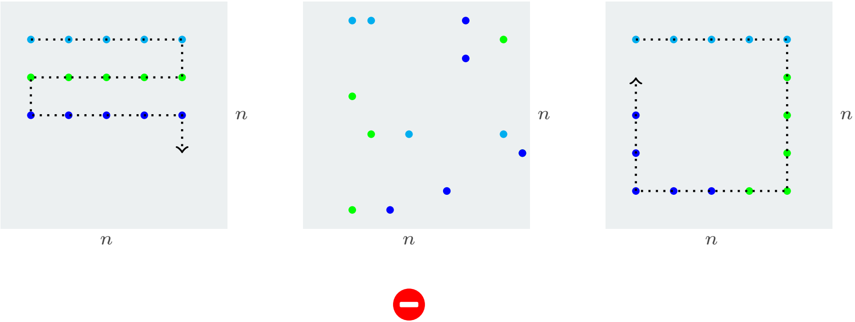
A fast acquisition needs few changes of patterns



Mirror galvanometer limitations: continuous trajectory & constant speed

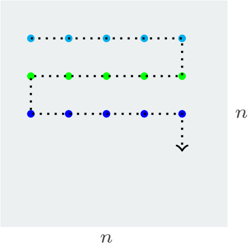
Constant sampling frequency to collect observations

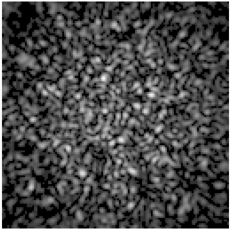
A fast acquisition needs few changes of patterns



What is the optimal distance between two repetitions of the same speckle?

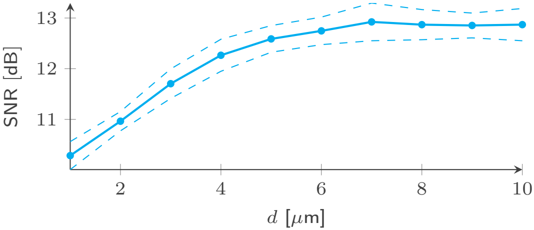
Distance d between two illumination patterns



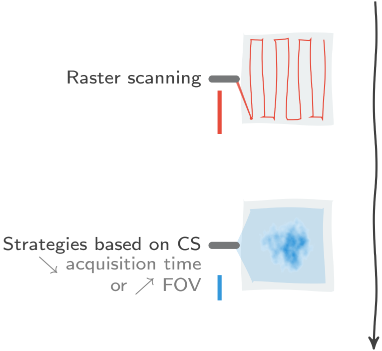


Factors

Depth
Laser wavelength
Single-core diameter



The lensless endoscope



Exploring and imaging cells in vivo

↗ Resolution   ↗ Imaging depth

S. Guérit, S. Sivankutty, C. Scotté, *et al.*, "Compressive sampling approach for image acquisition with lensless endoscope," *ArXiv*, 2018