Applications of PCA and low-rank plus sparse decompositions in high-contrast Exoplanet imaging

Carlos A. Gómez, ULG

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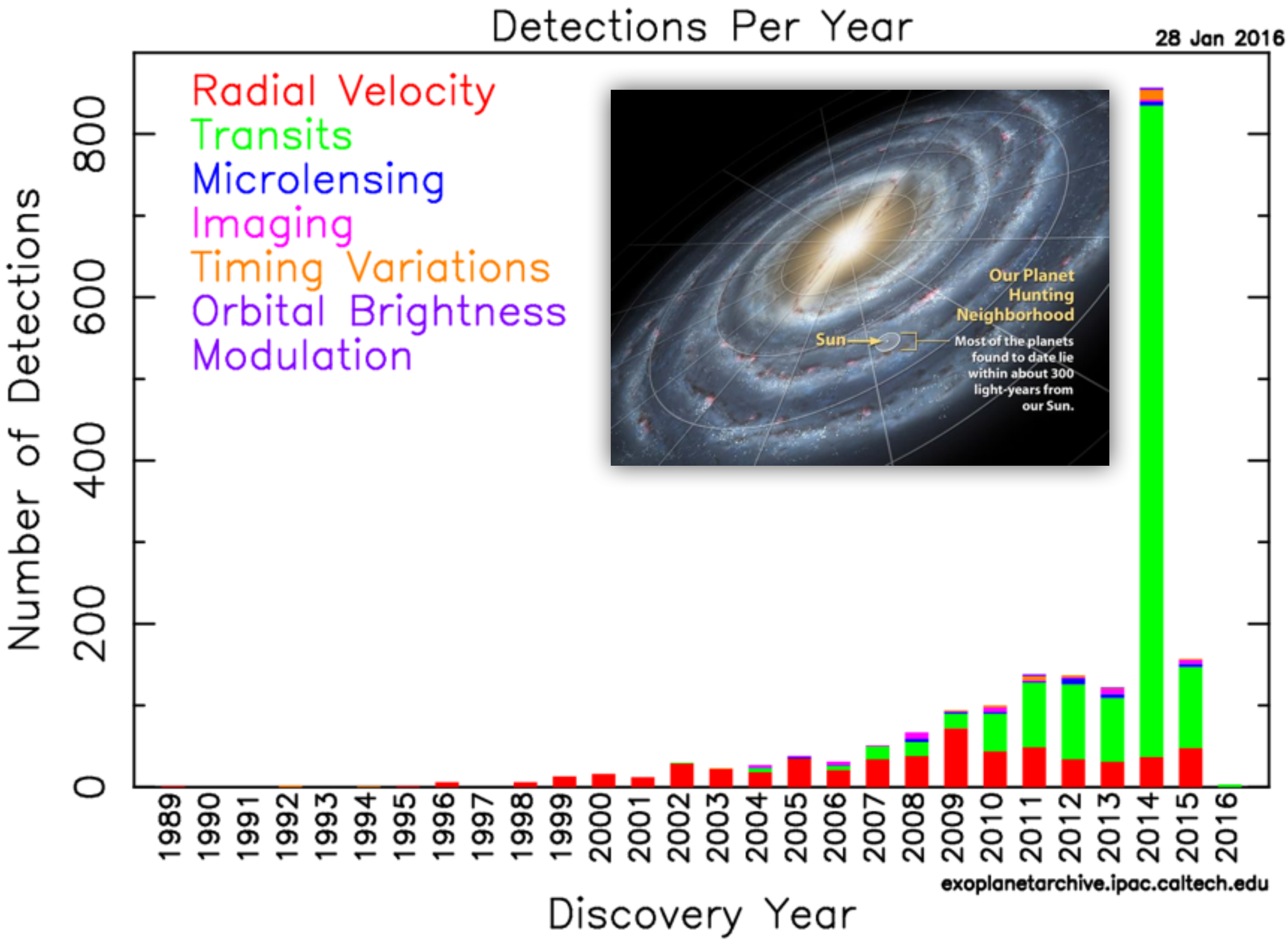
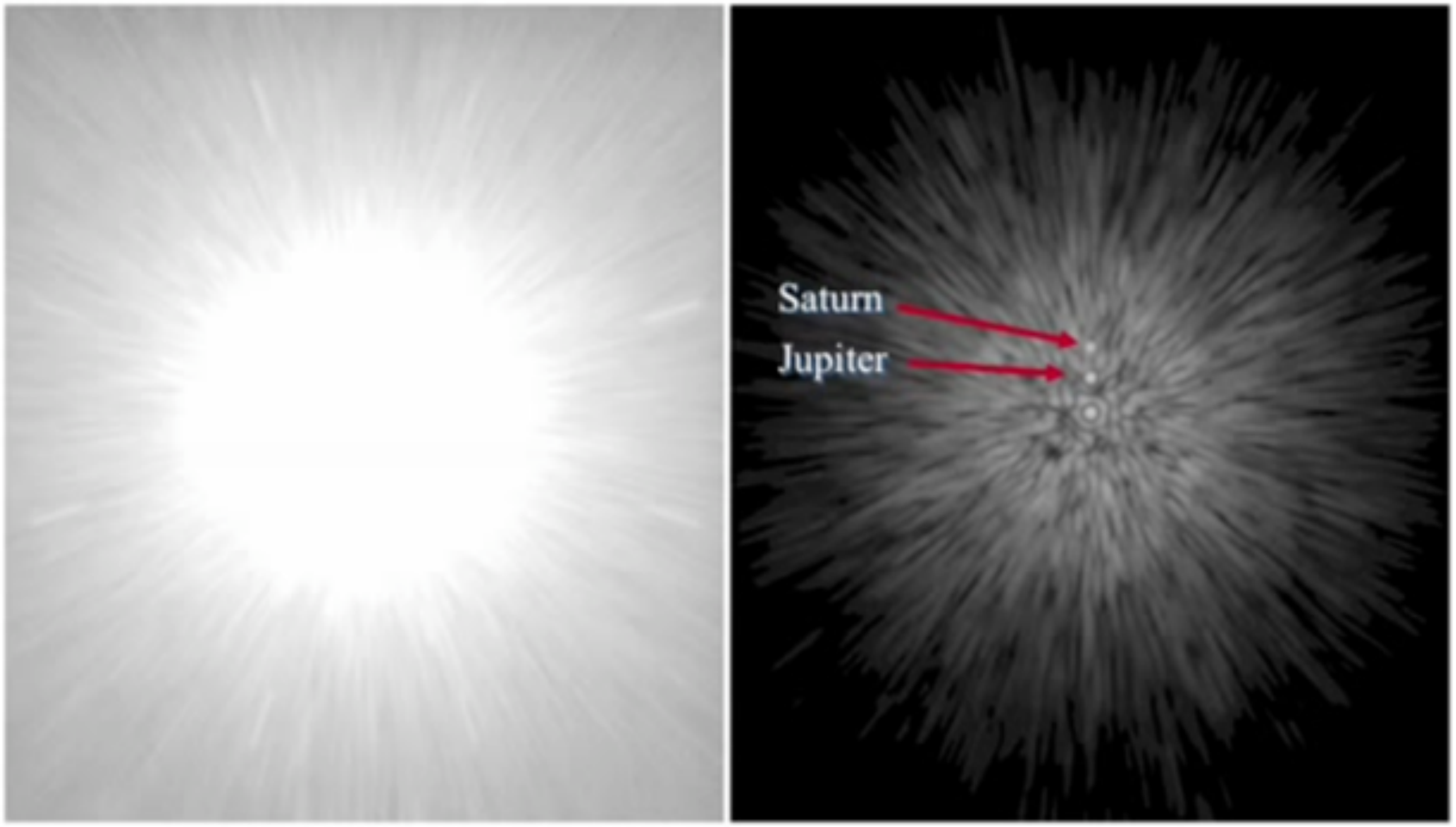
Exoplanets



VORTEX

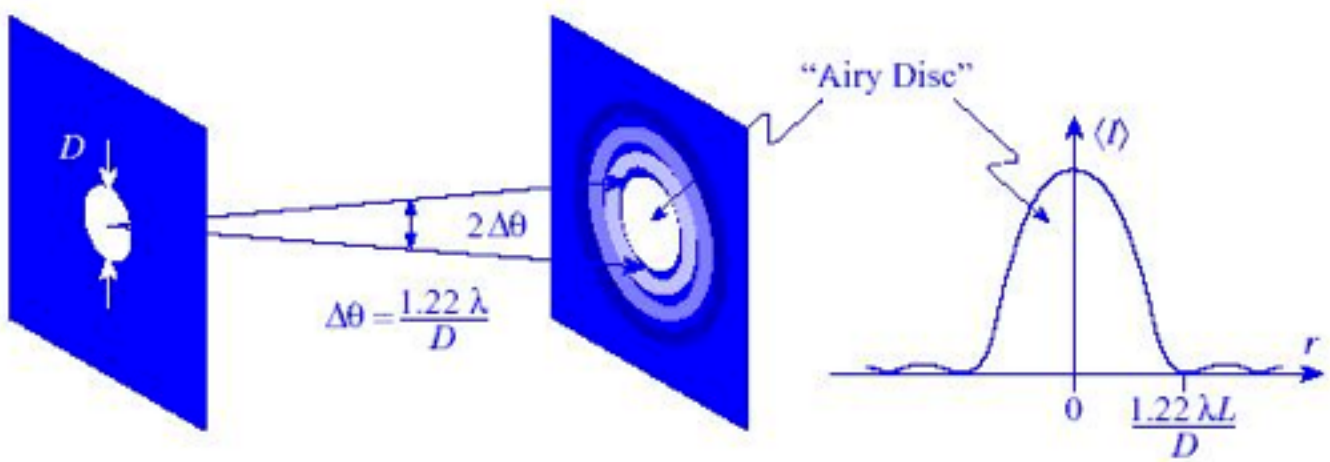
Challenges of direct imaging

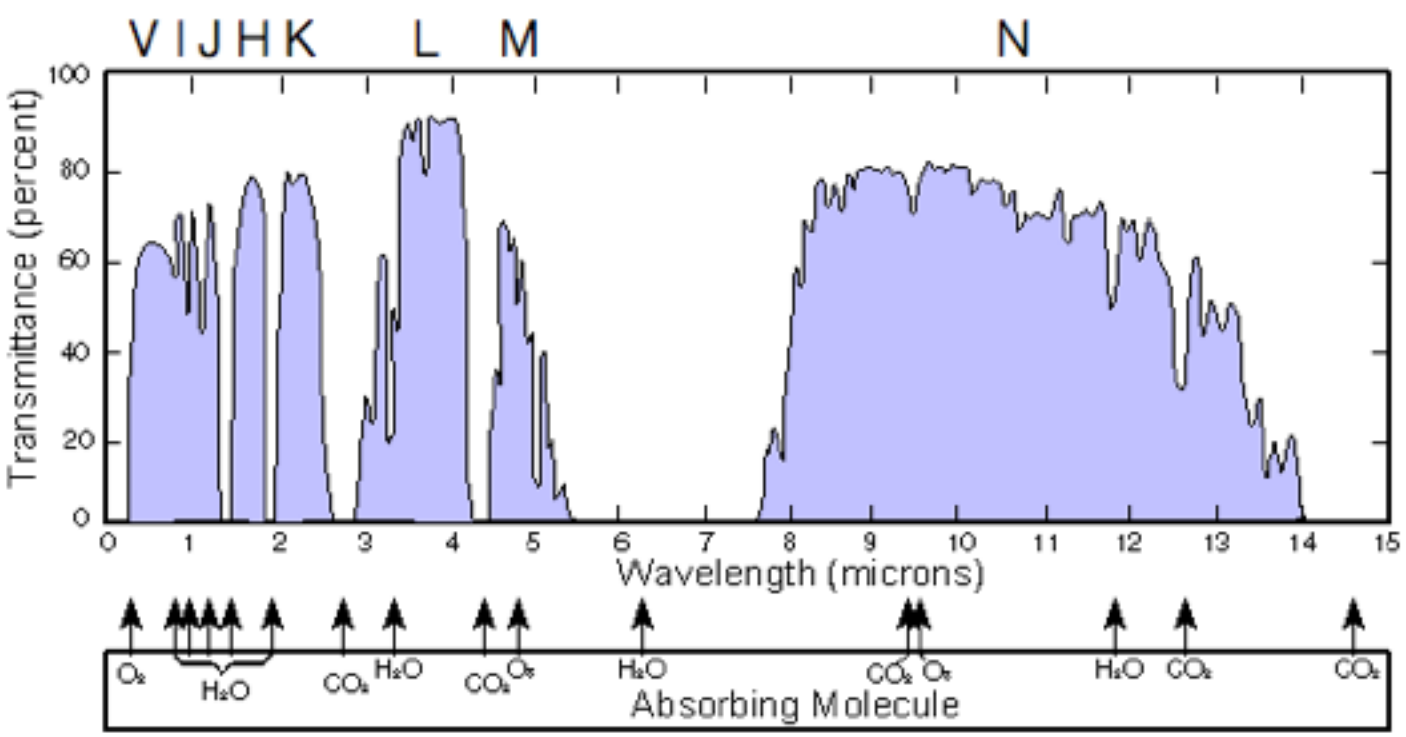


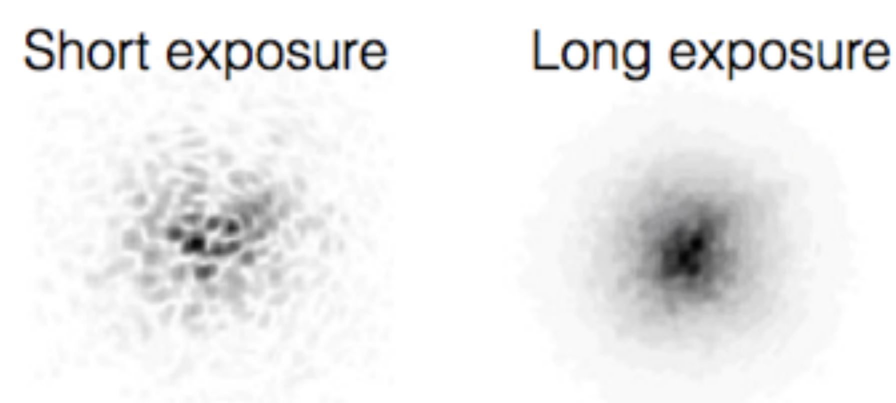
- Contrast
- Angular separation
- Speckles

Credits: Rebecca Oppenheimer

Imaging from the ground

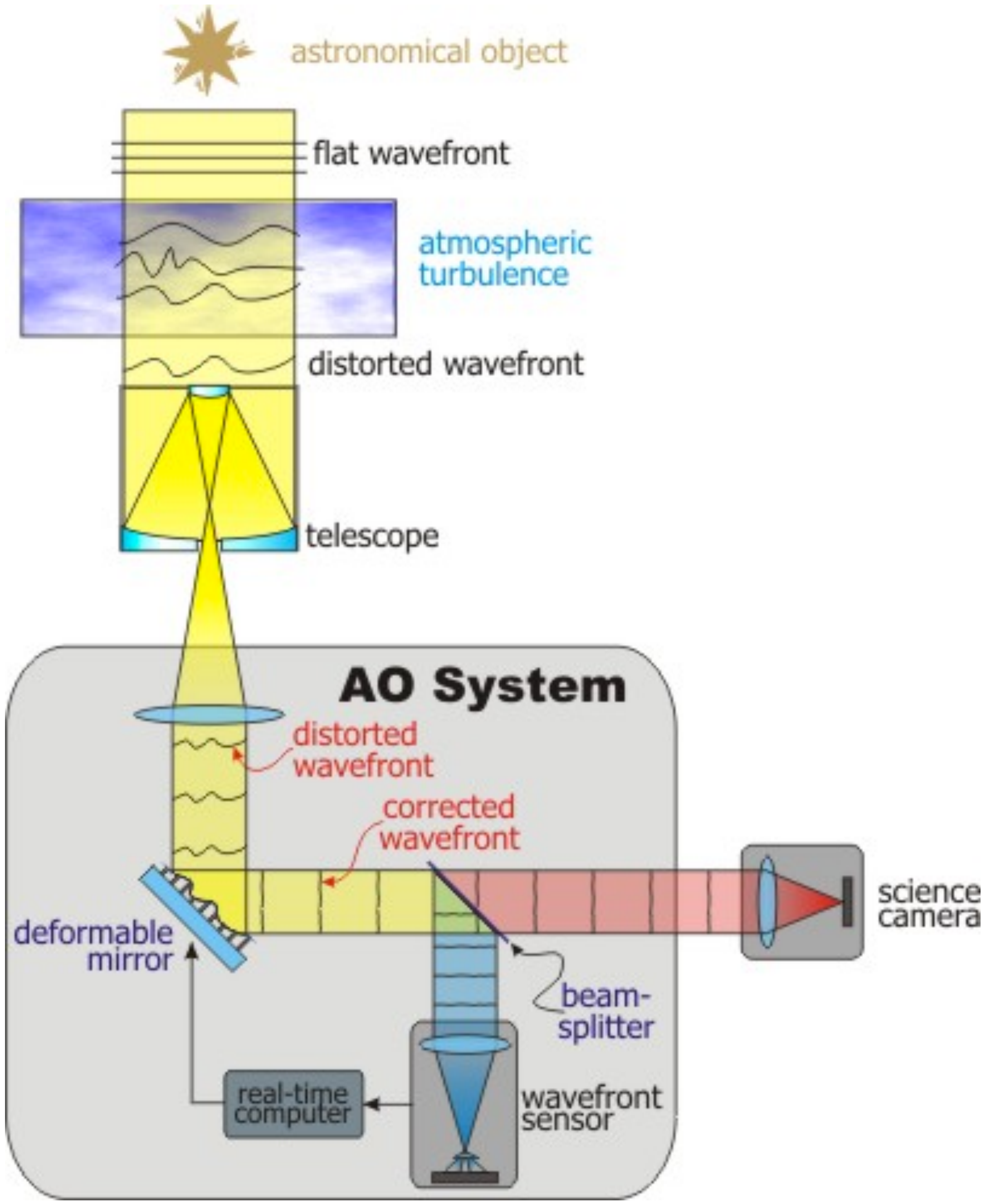




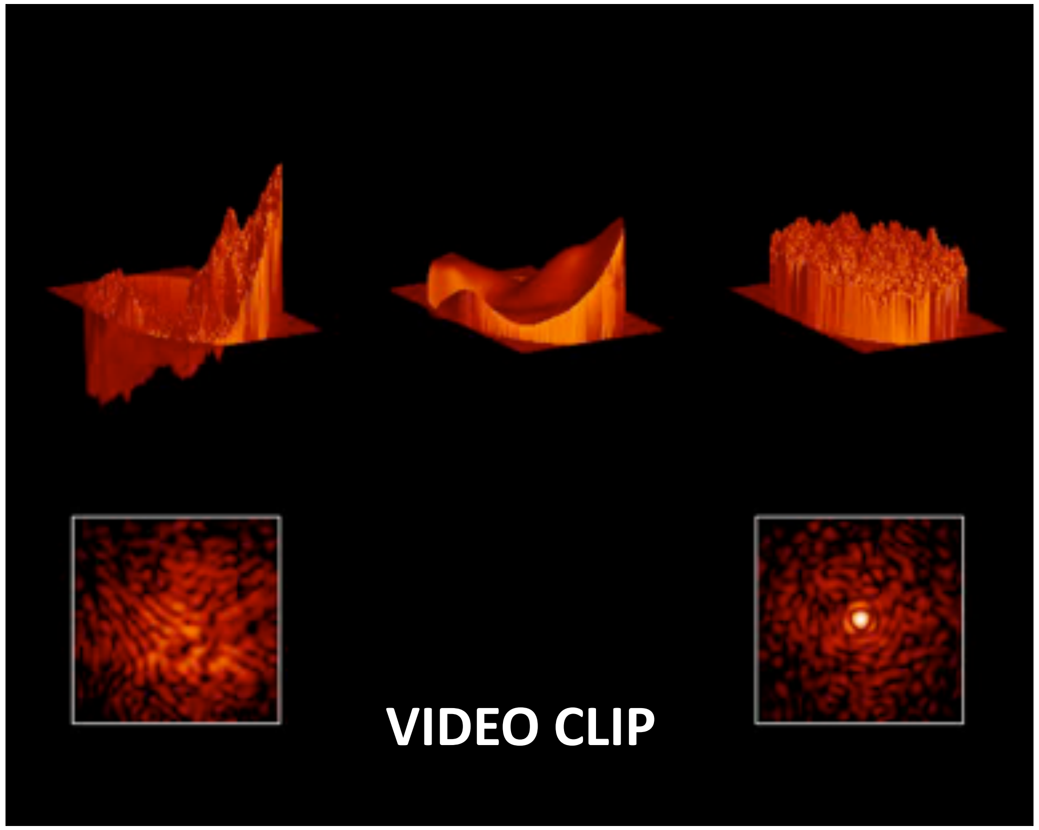


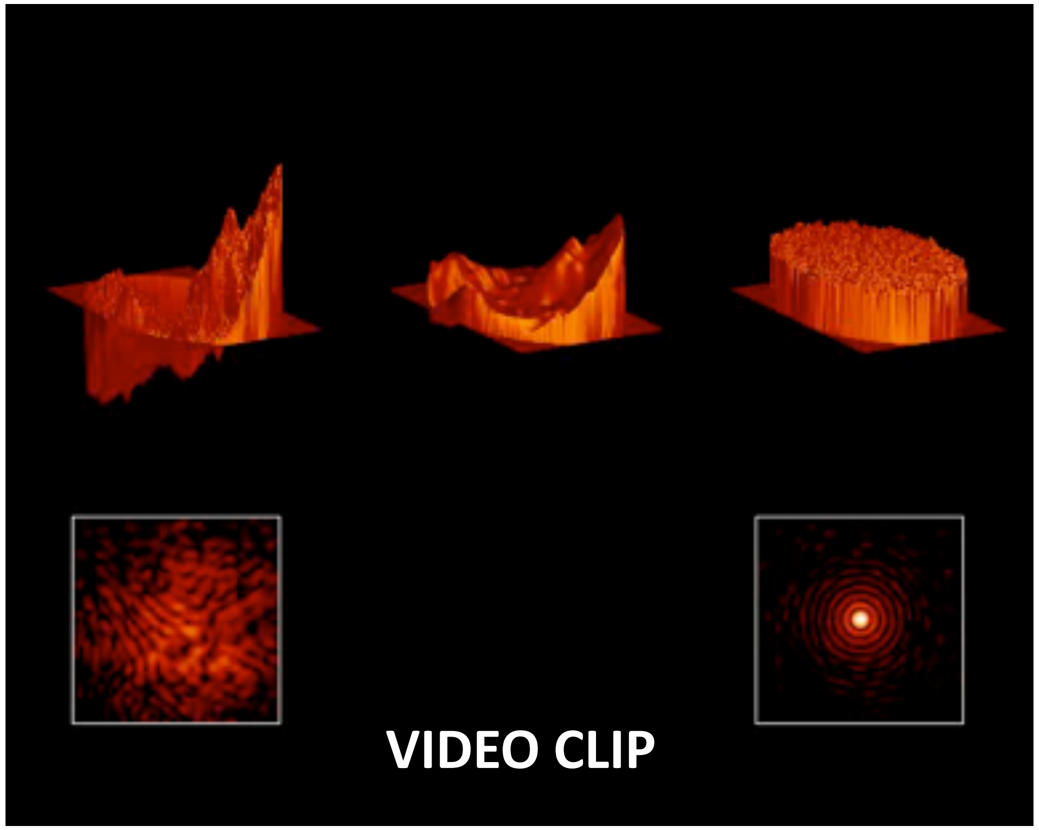
- Atmospheric turbulence distortion

Adaptive optics



Credits: Stuart Littlefair

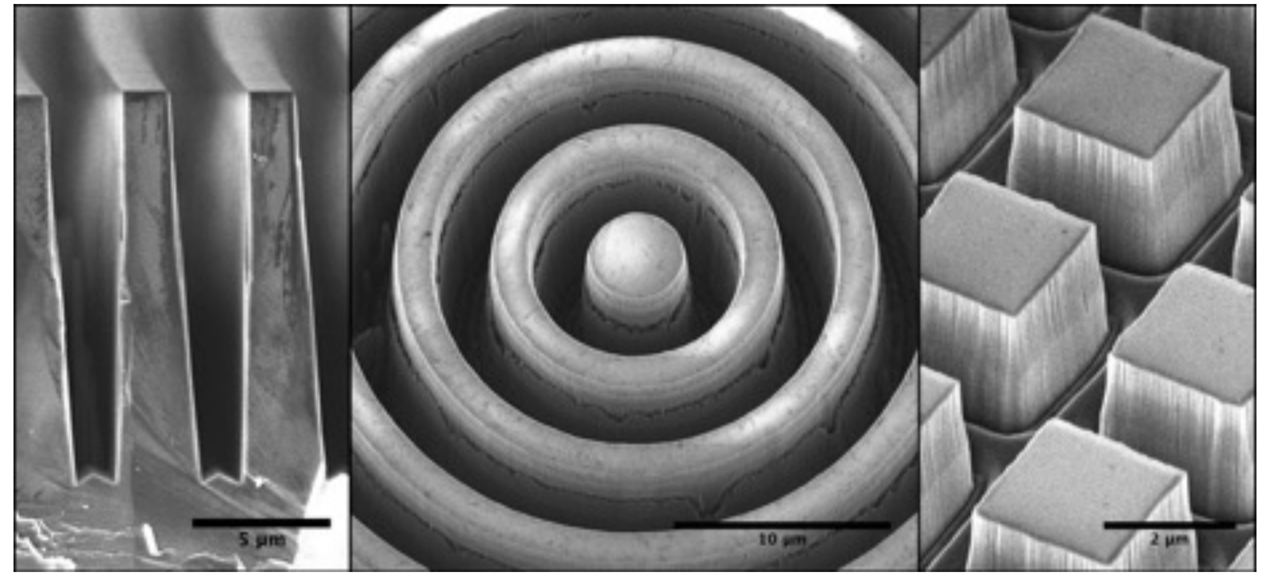




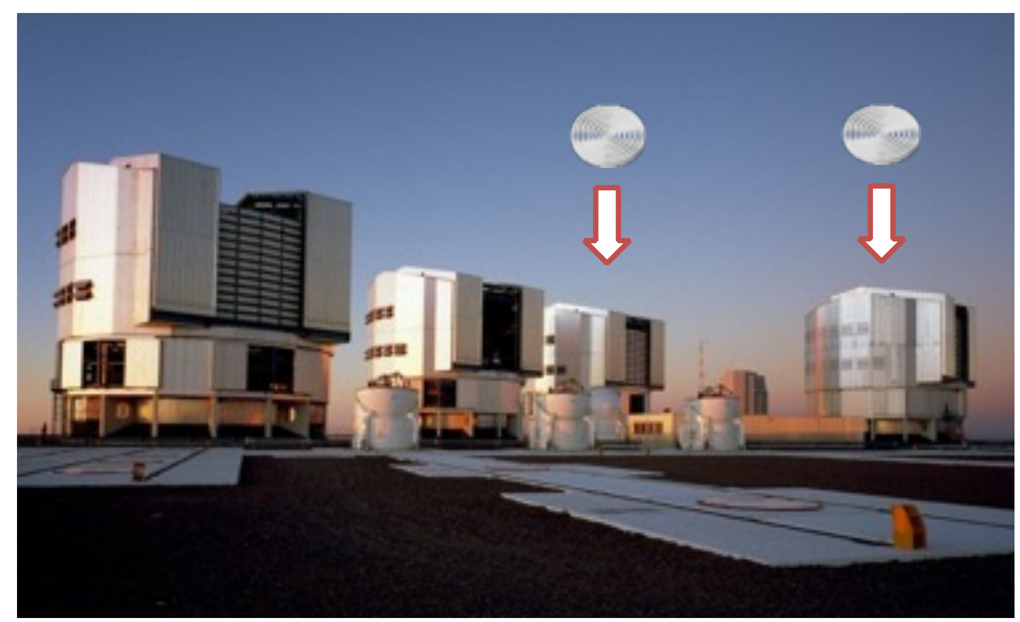
Credits: LYOT project

Coronagraphy

- Coronagraphs used to mask the star light
- Vortex coronagraph
  - Diamond substrate
  - Optical / IR transmission
  - Mechanical / thermal properties
  - Diamond etching (Uppsala)
  - Installed at largest ground based telescopes



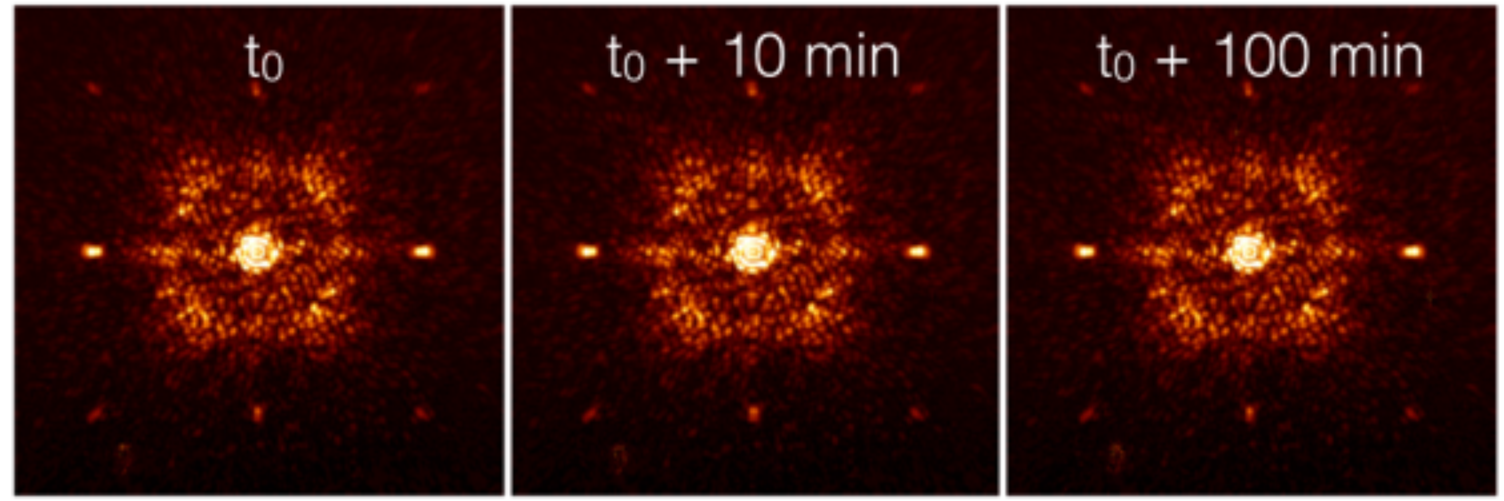
Delacroix et al. 2014

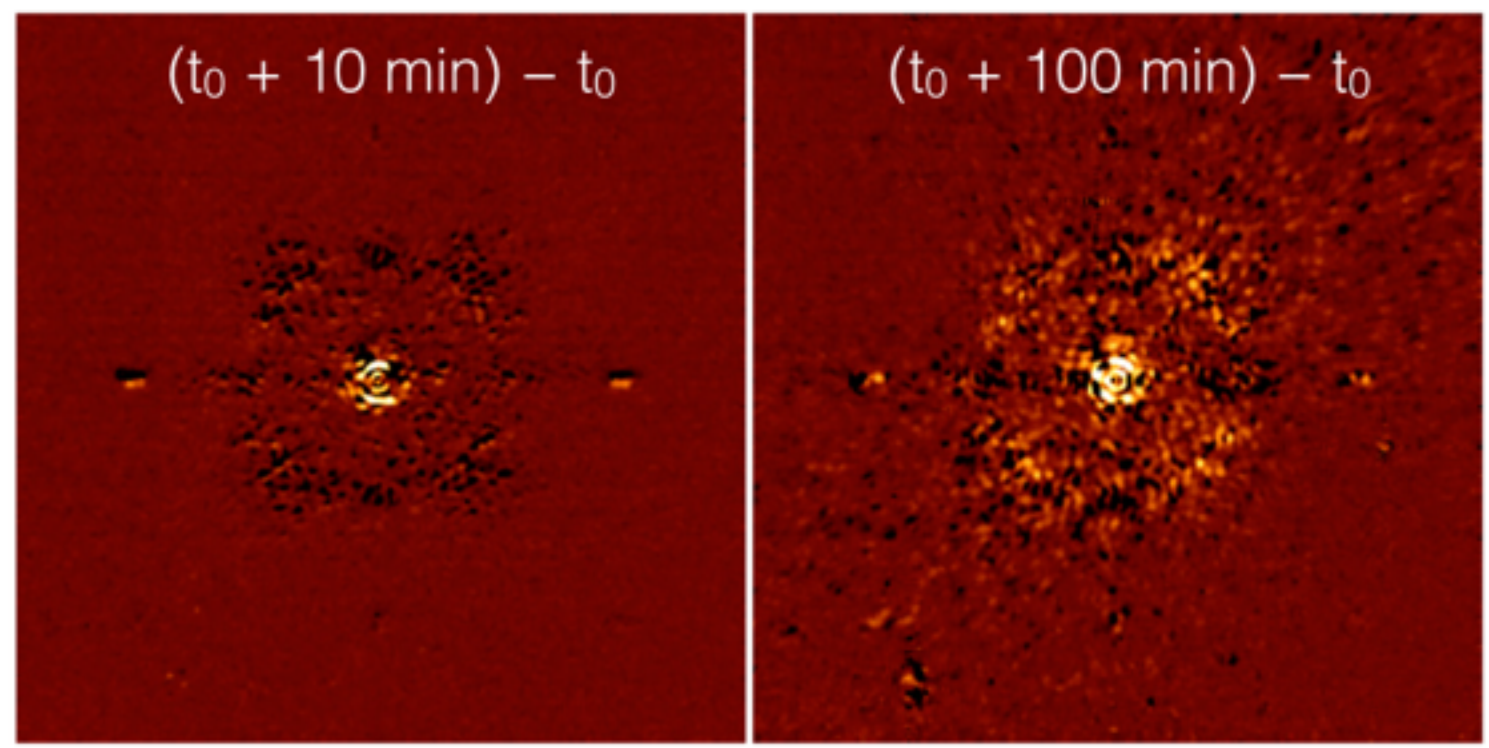


Mawet et al. 2013

Speckles

Aime et al 2014





Two kinds: short lived atmospheric speckles and quasi-static speckles originating from the telescope/instrument (which limits AO exposures)

$$p_{MR}(I, I_c, I_s) = \frac{1}{I_s} \exp\left(-\frac{I + I_c}{I_s}\right) I_o\left(\frac{2\sqrt{I I_c}}{I_s}\right)$$

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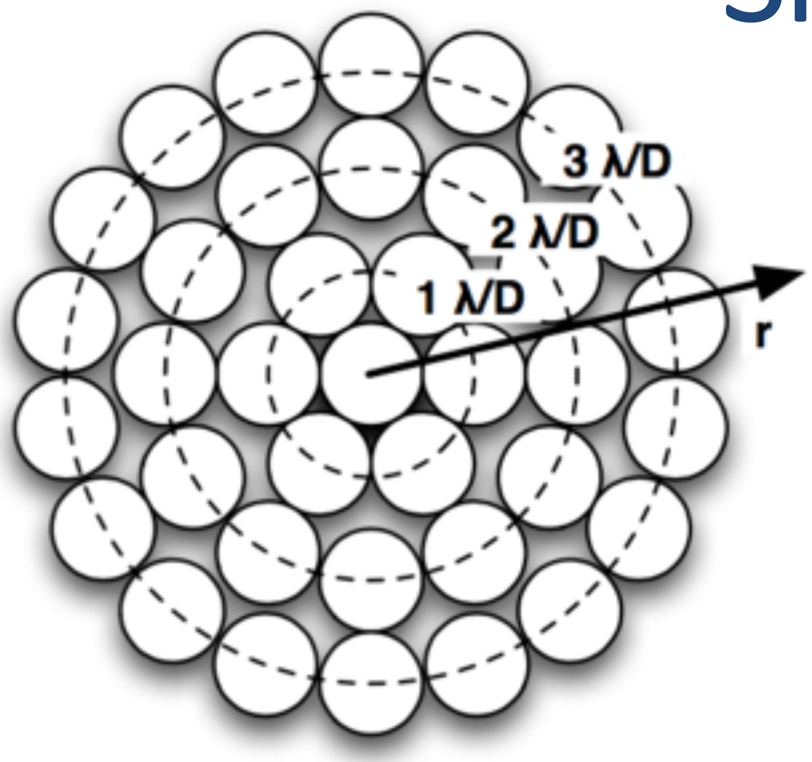
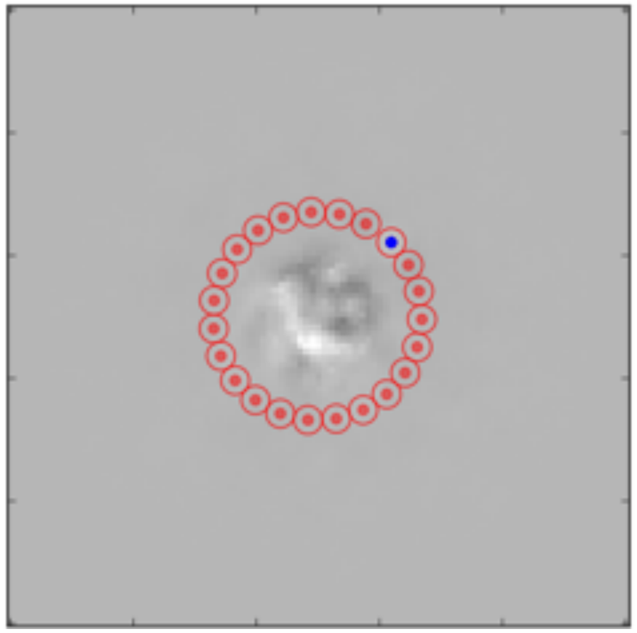
Signal-to-noise





$$SNR = \frac{\overline{x}_1 - \overline{x}_2}{s_2 \sqrt{1 + \frac{1}{n_2}}}$$

When doing small angle high-contrast imaging we are in the small sample statistics regime (Mawet et al. 2014).

A two sample t-test is used to test whether the intensity of a given resolution element is statistically different from background ones.

This imposes a penalty factor on detection threshold (10 at 1λ/D, 2 at 2λ/D for a 5σ CL).

Problematic metric. Best we have.

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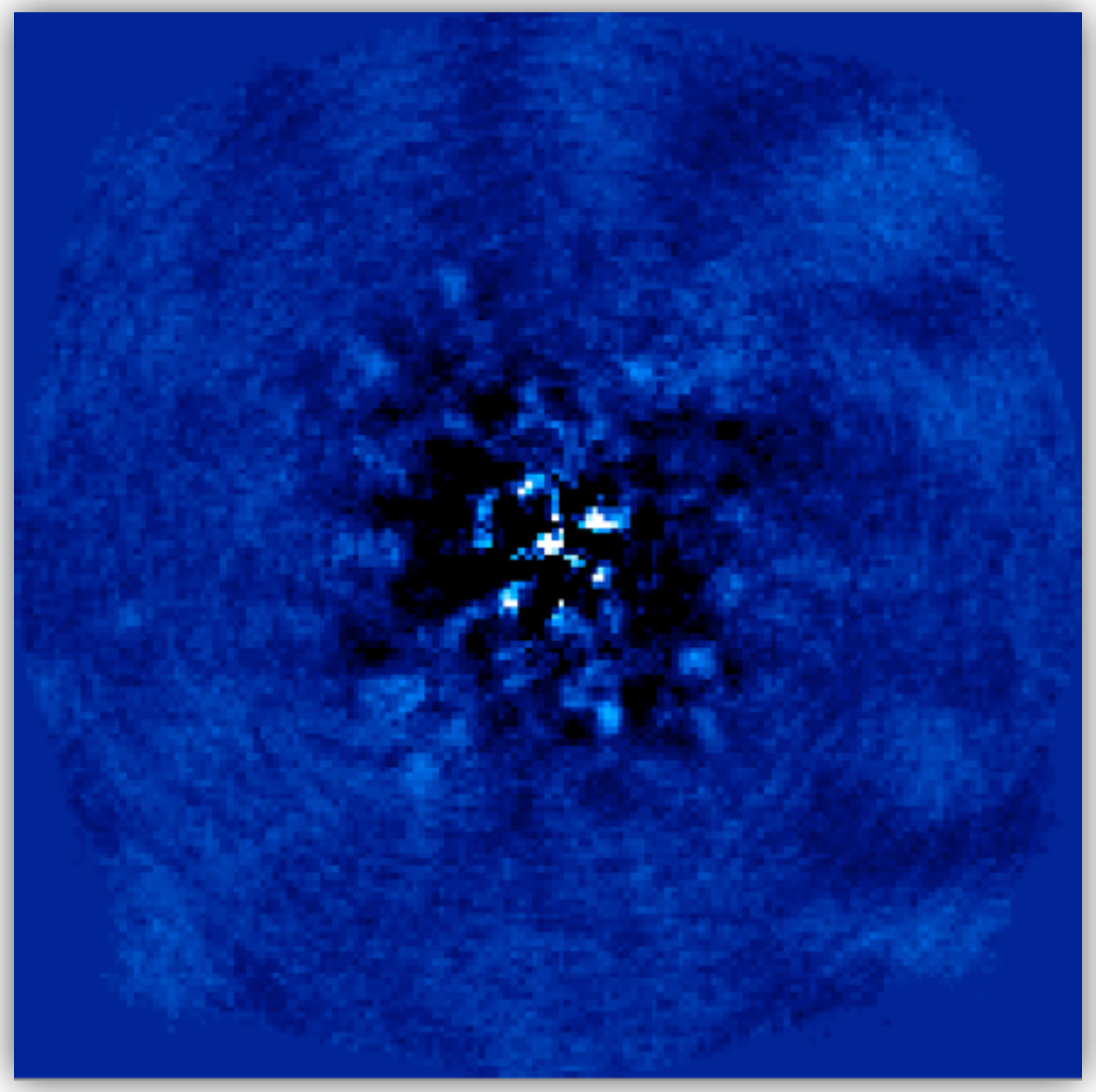
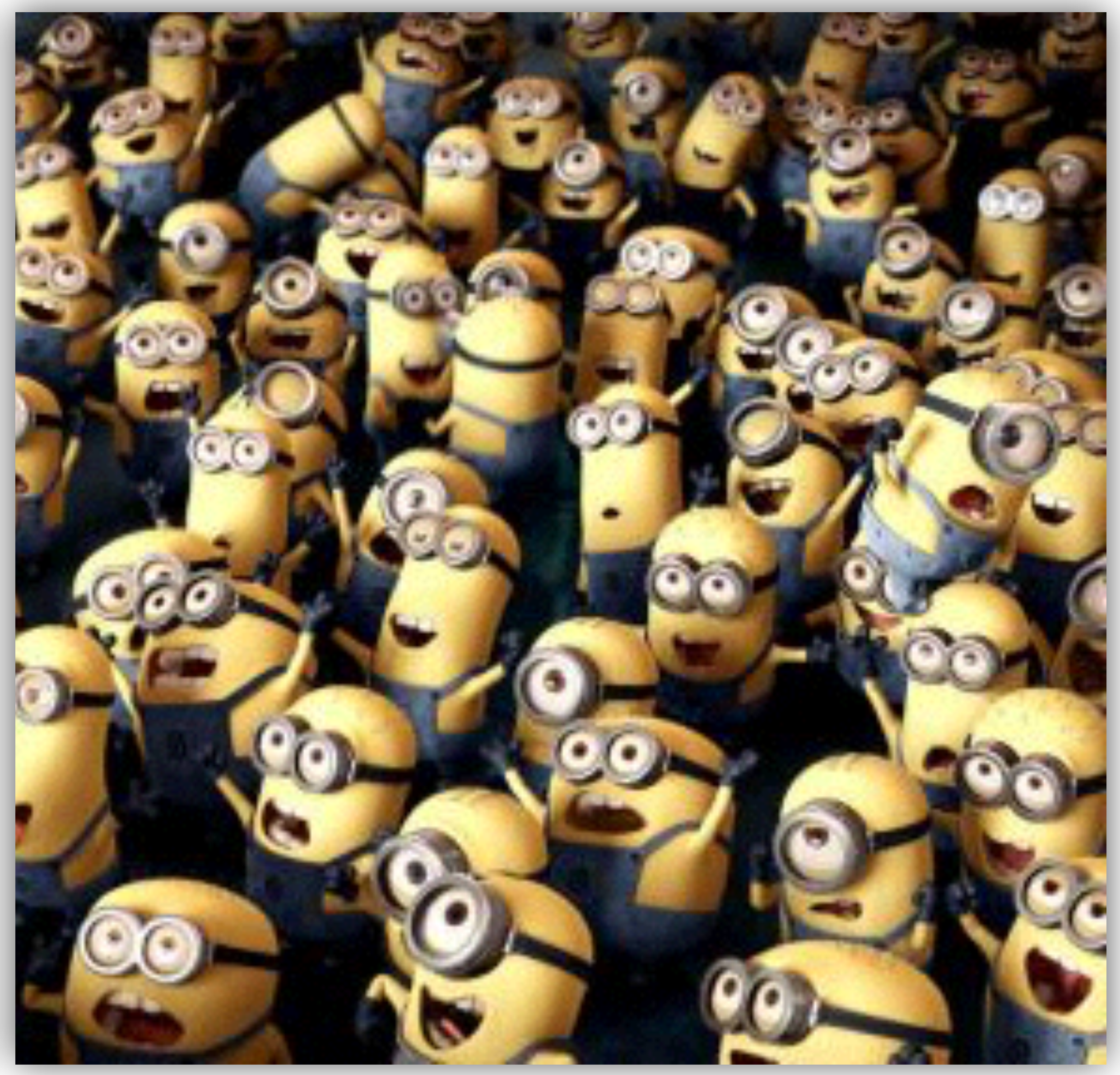
Summary in one image





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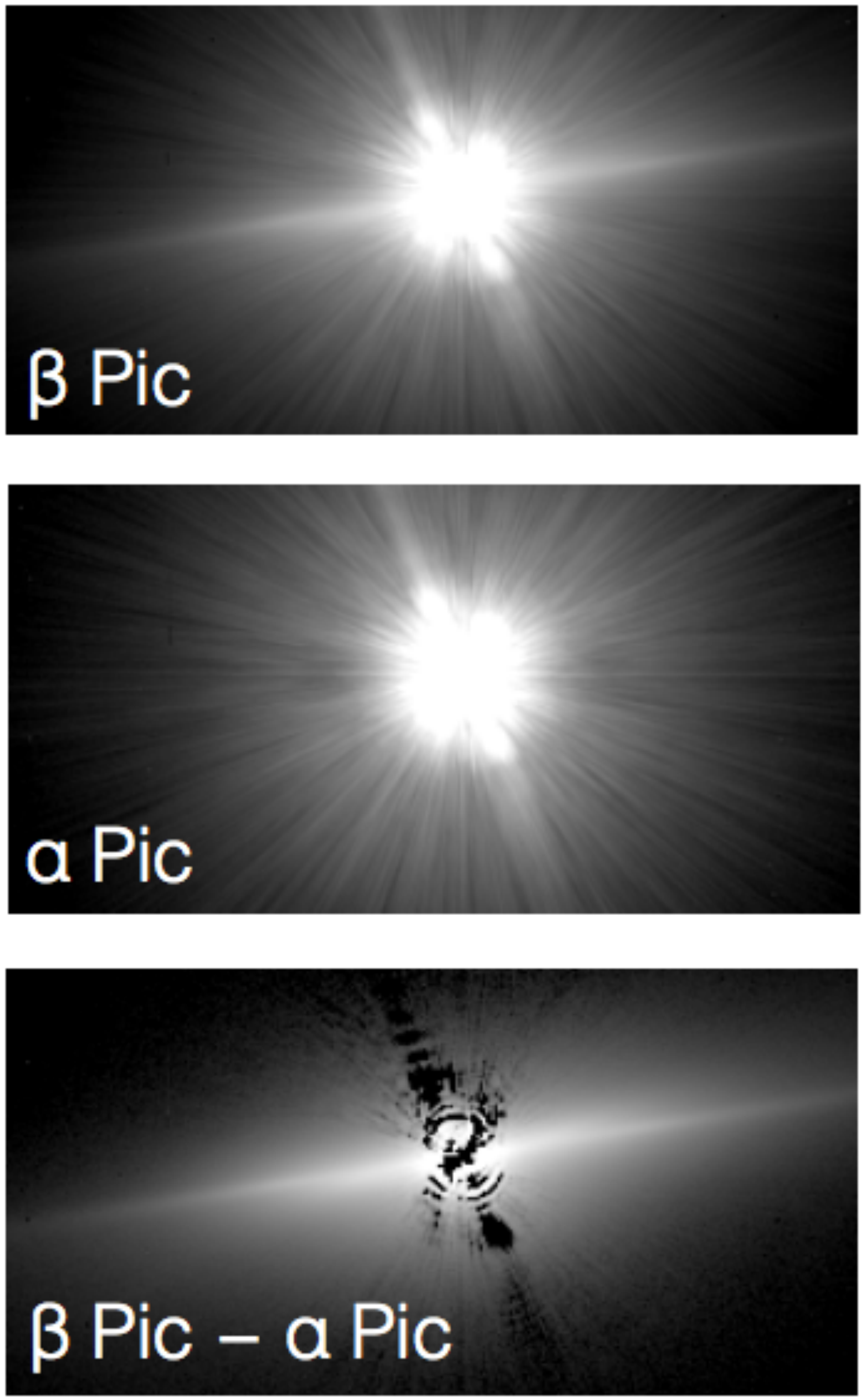
Differential imaging

Reference star Differential Imaging (RDI)



Credits: HST

PROS:

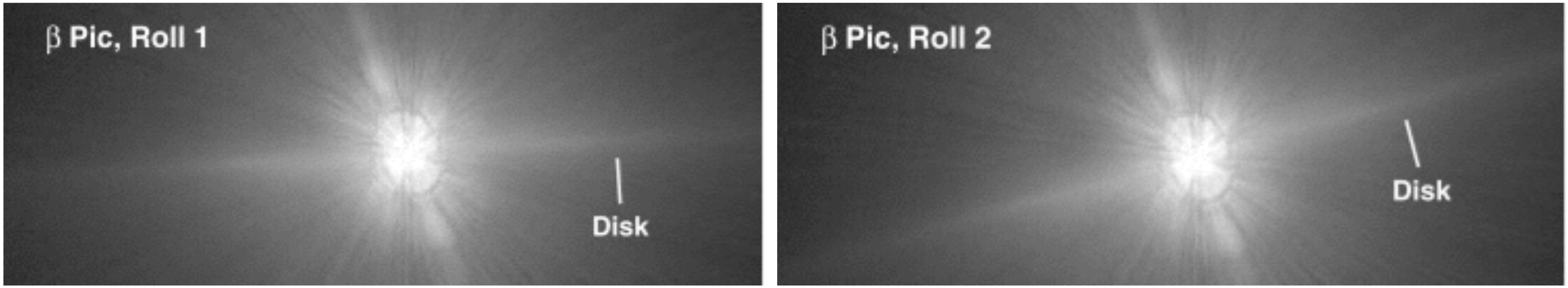
- simple idea, we can use archival data
- access to small angles (~ λ/D)

CONS:

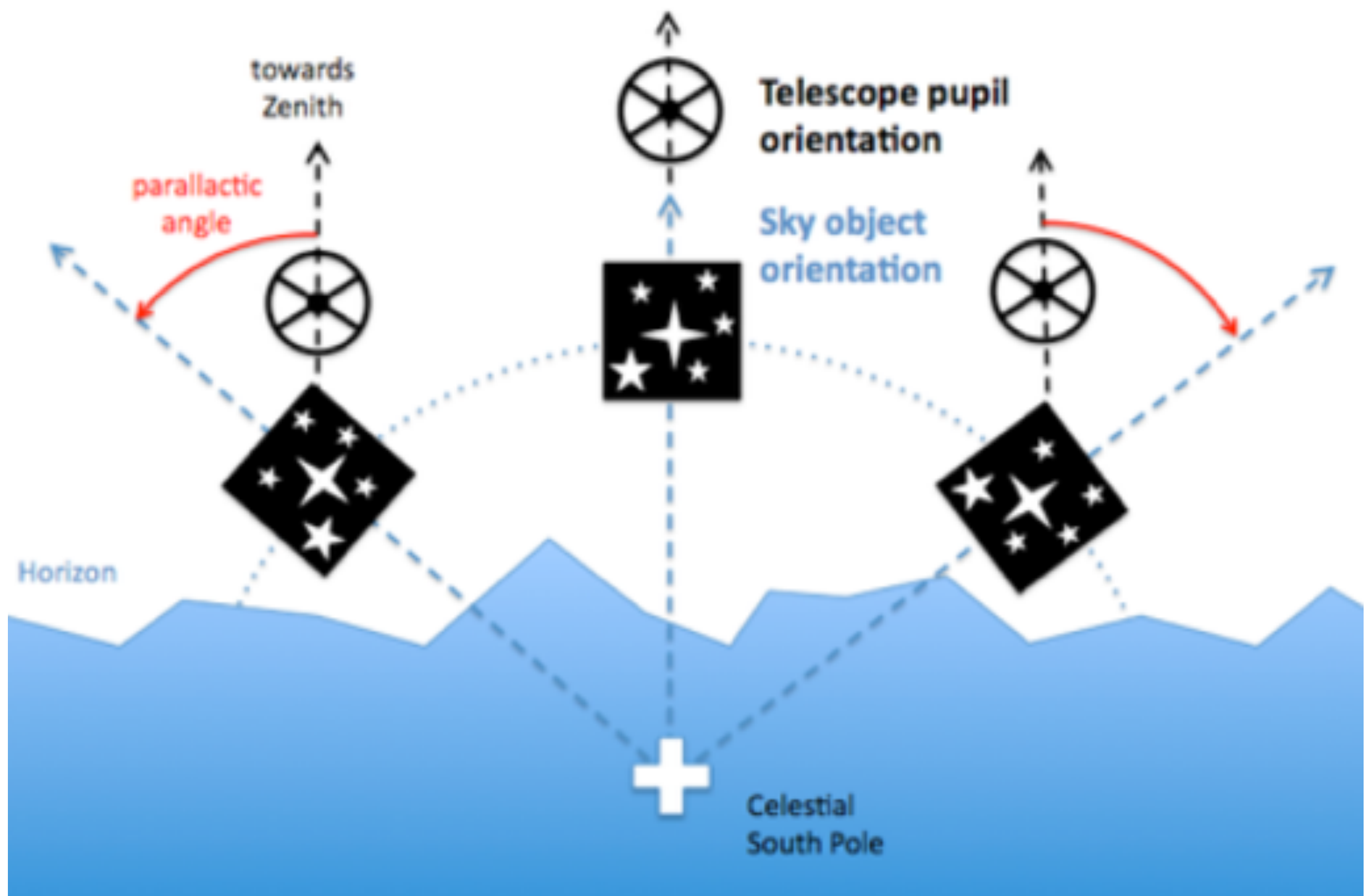
- hard execution: no perfect reference star (image)
- atmospheric conditions change
- telescope/instrument aberrations change with time
- PSF and speckle pattern changes

Differential imaging

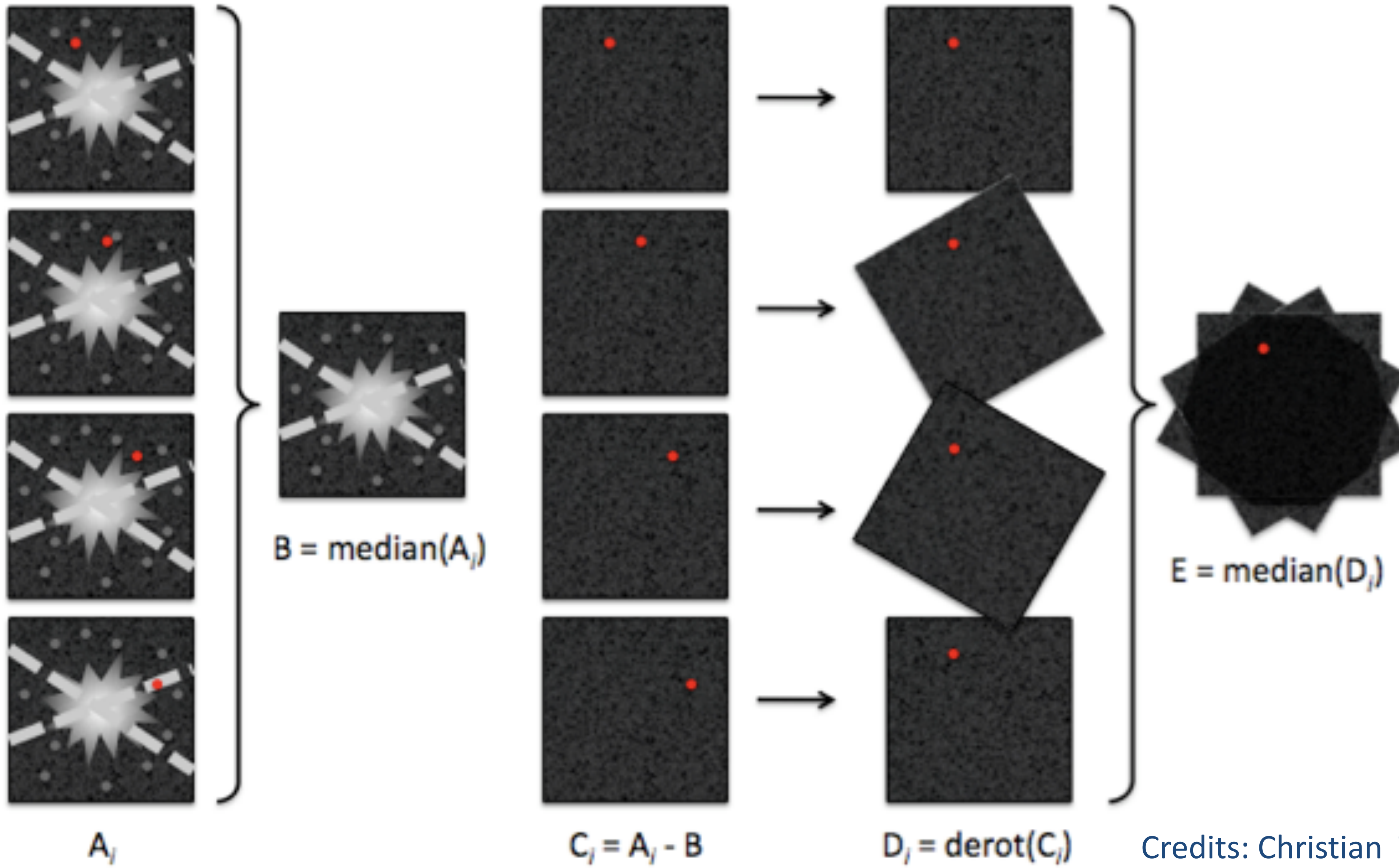
Angular Differential Imaging (ADI)



Credits: HST



Credits: Christian Thalmann



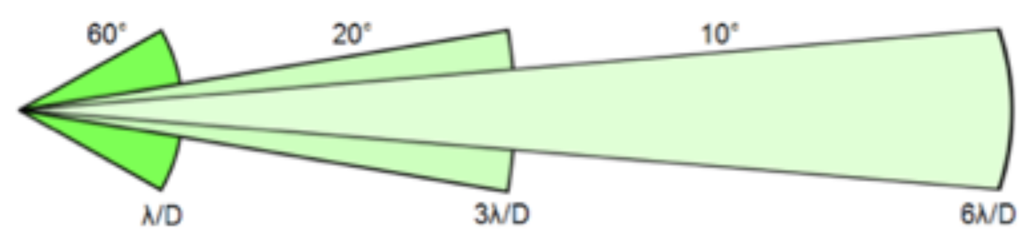
Credits: Christian Thalmann

PROS:

- observations of the same target (any star)

CONS:

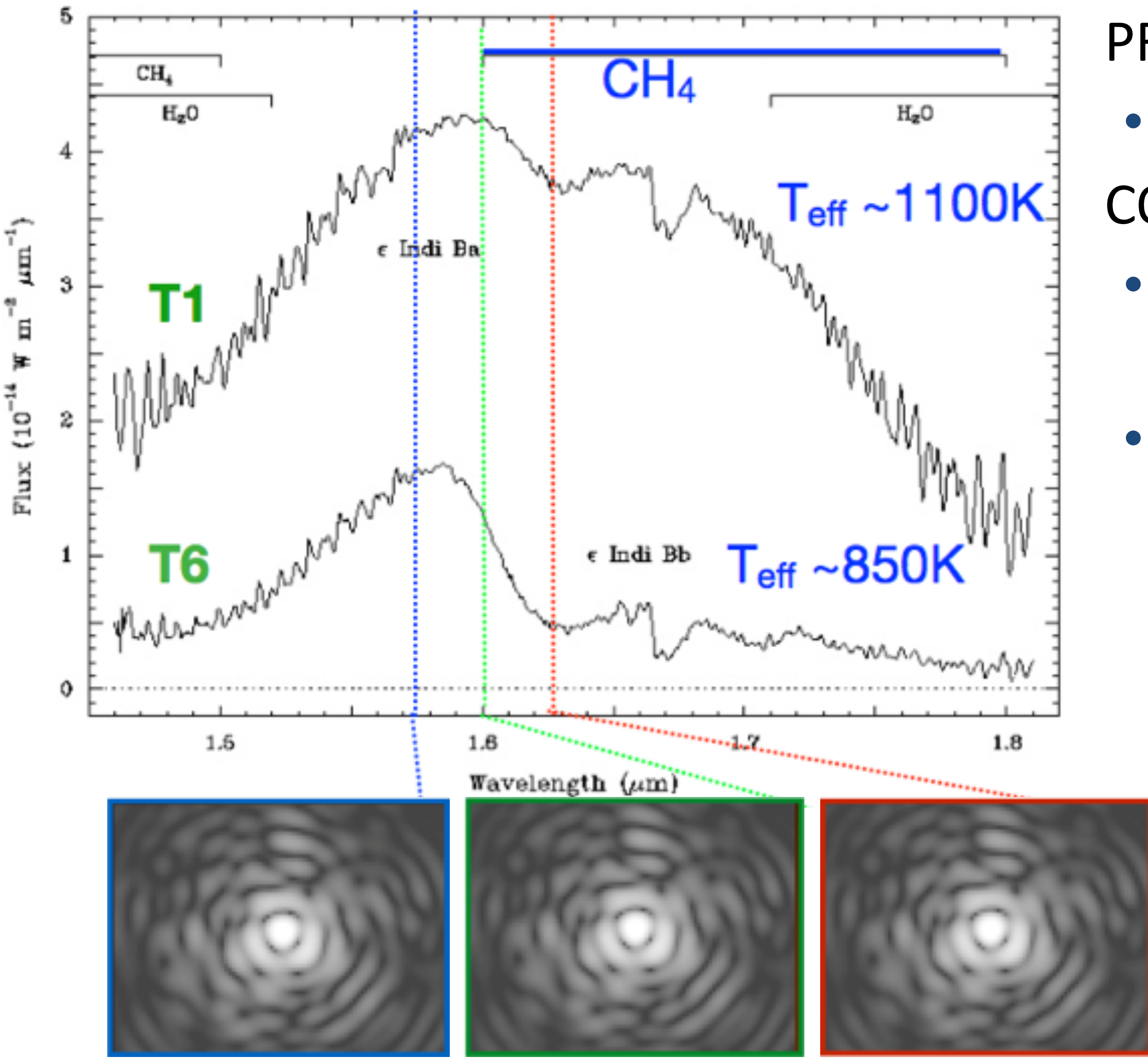
- speckle patter evolves with time
- limited inner working angle (at least 1 λ/D rotation)



Differential imaging

Spectral Differential Imaging (SDI)

McCaughrean et al. 2003



Credits: Olivier Absil

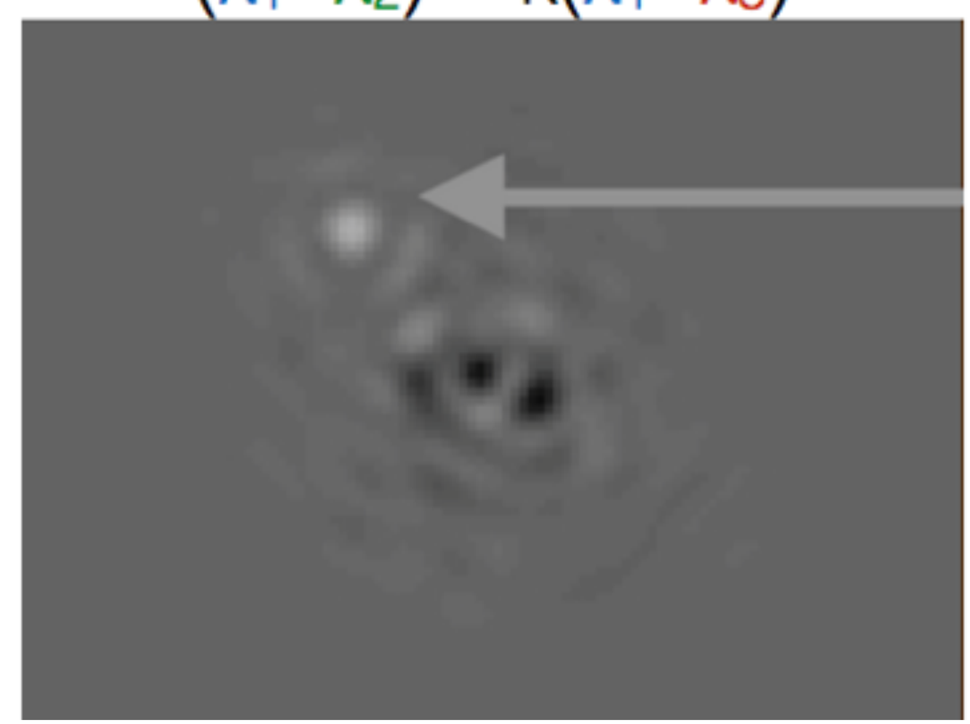
PROS:

- access to small angles (~ λ/D)

CONS:

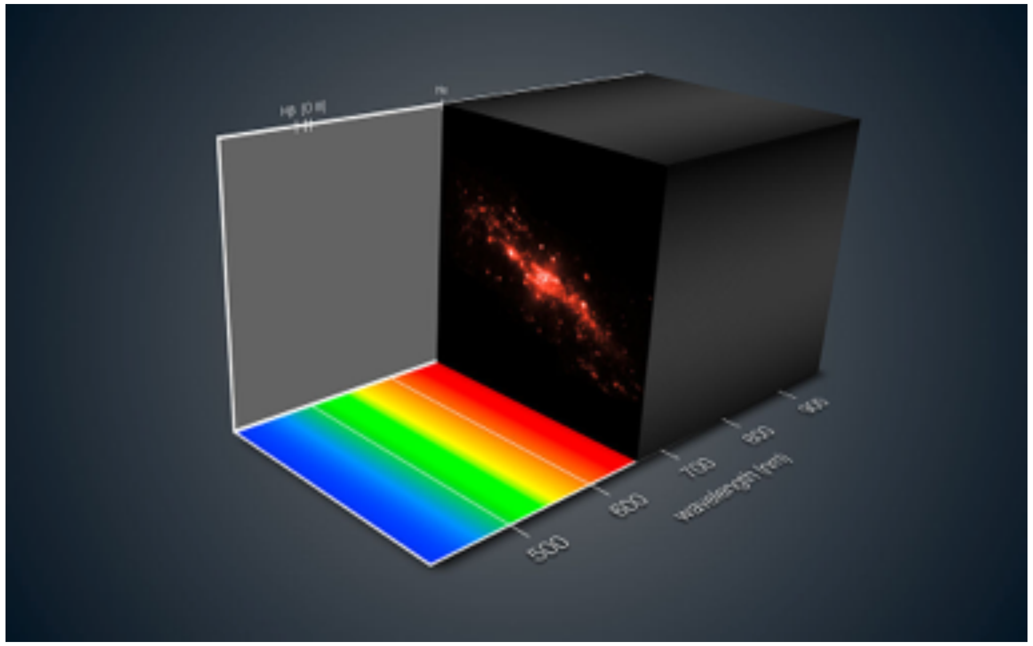
- need of strong molecular absorption
- differential aberrations between the 3 optical paths

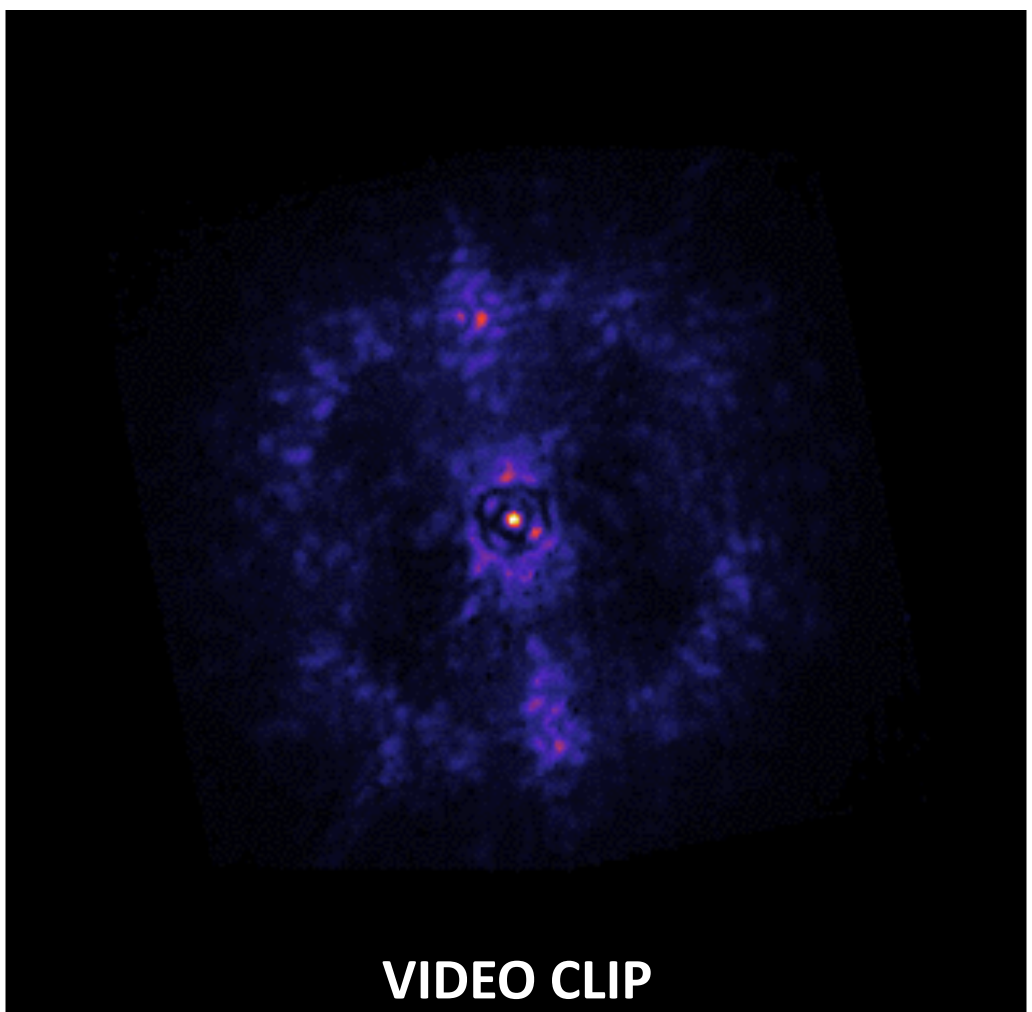
$$(\lambda_1-\lambda_2) - k(\lambda_1-\lambda_3)$$



Differential imaging

Spectral Differential Imaging (SDI) - multiple spectral channels





Integral Field Spectrograph (IFS)

Diffraction and speckle pattern scale as a function of the wavelength. Exoplanet remains fixed

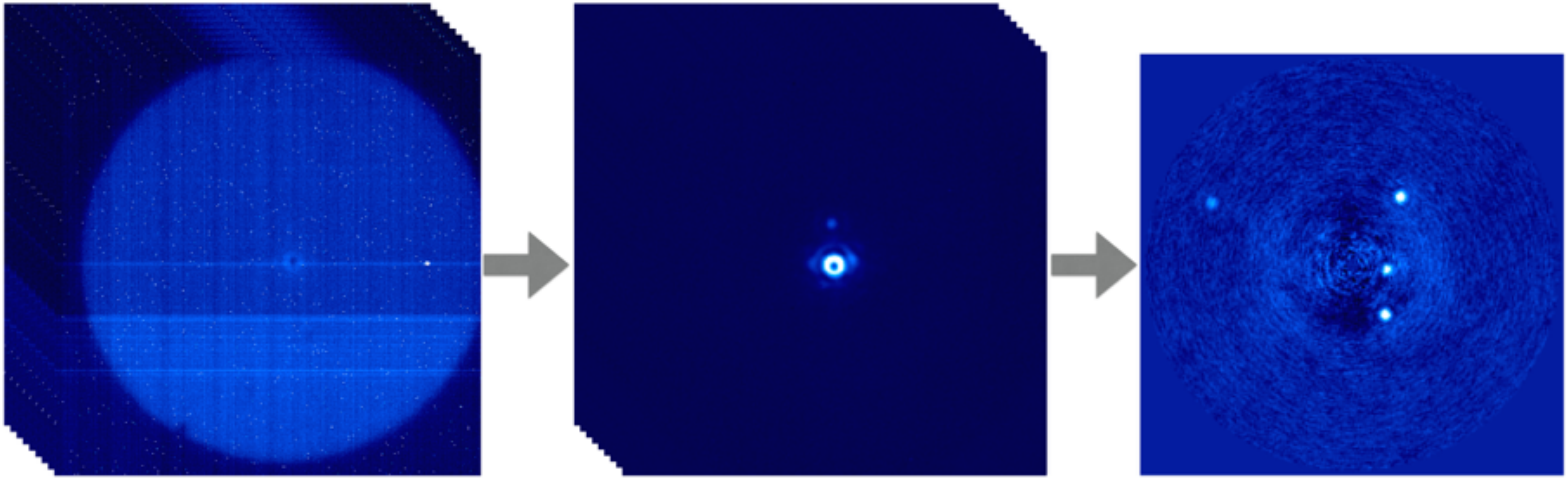
PROS:

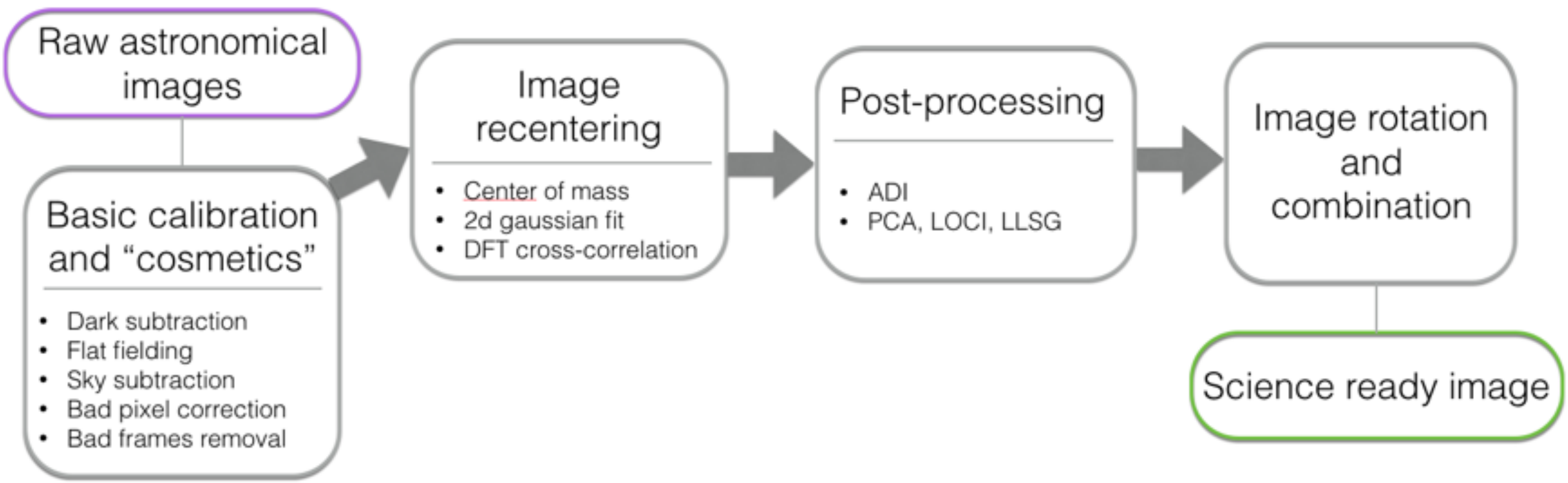
- we get the spectrum of the planet (detection+characterization)

CONS:

- speckles not constant over λ
- limited inner an outer working angles (depending on λ range)

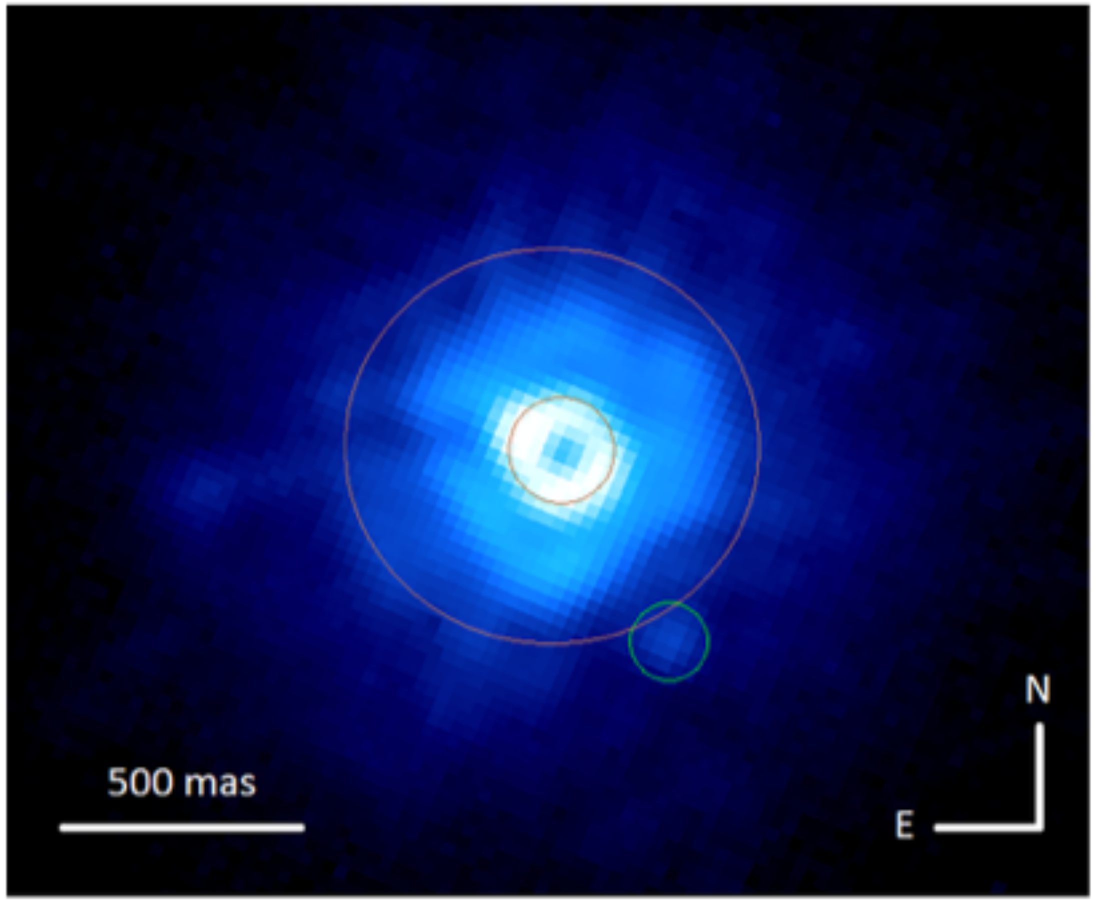
Image processing



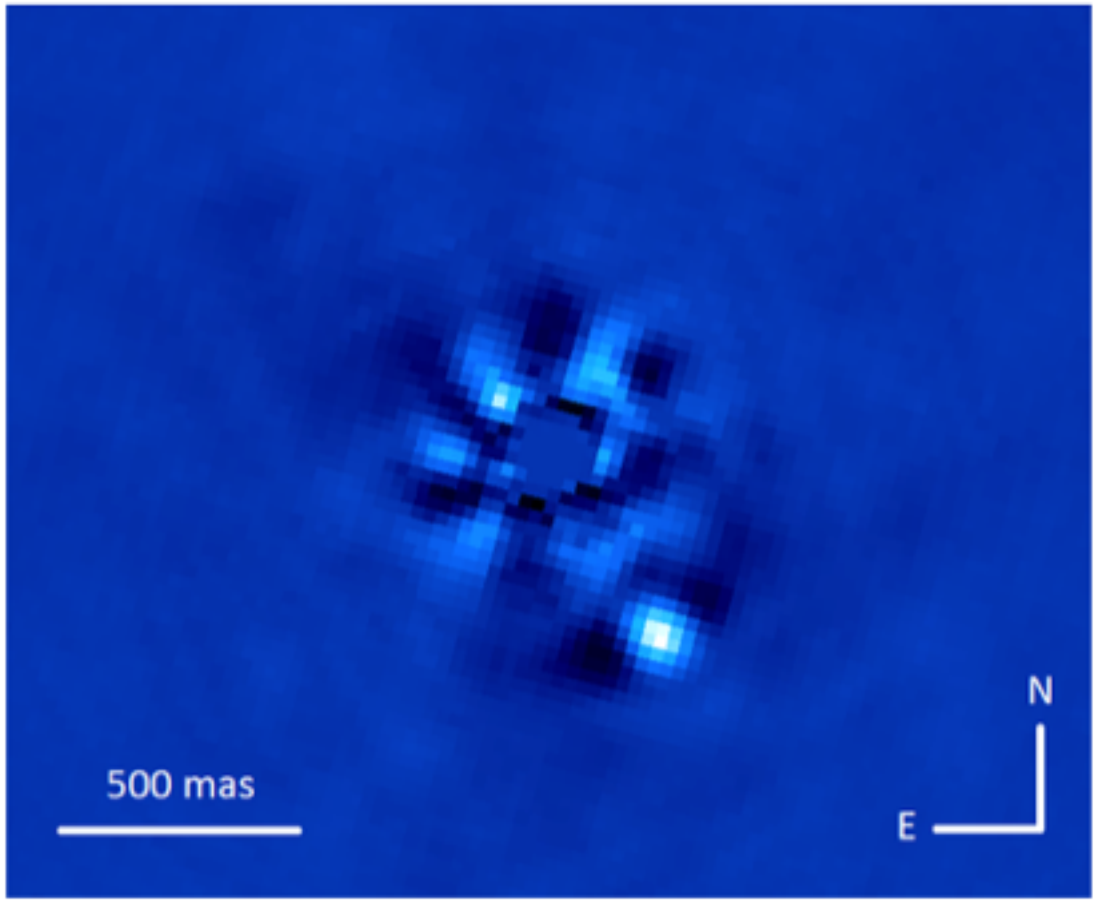


Example images

Raw image



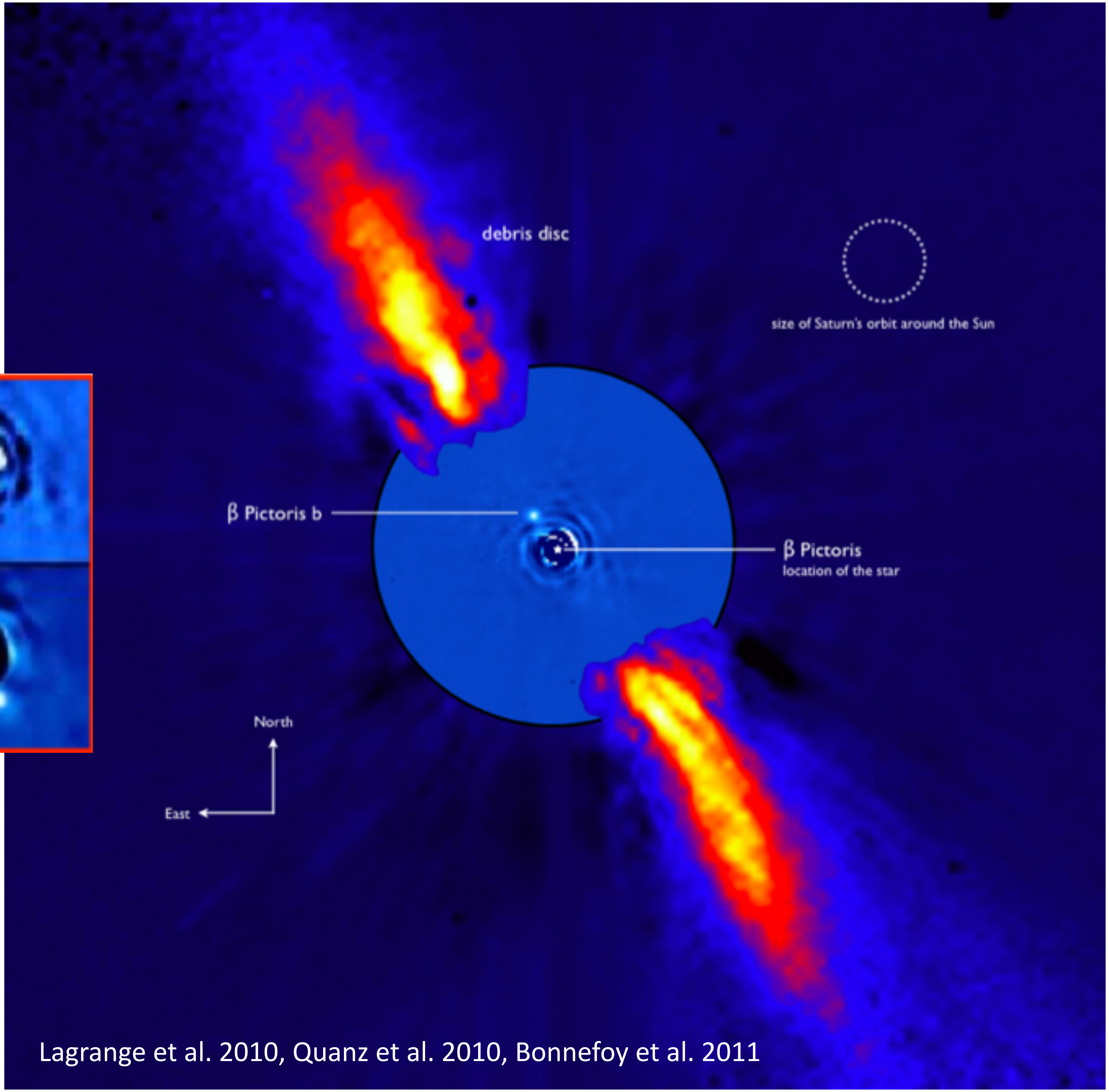
Post-processed (ADI+PCA)

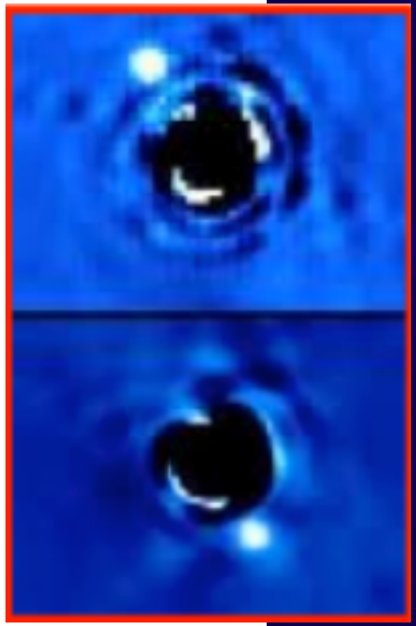


Absil et al. 2013

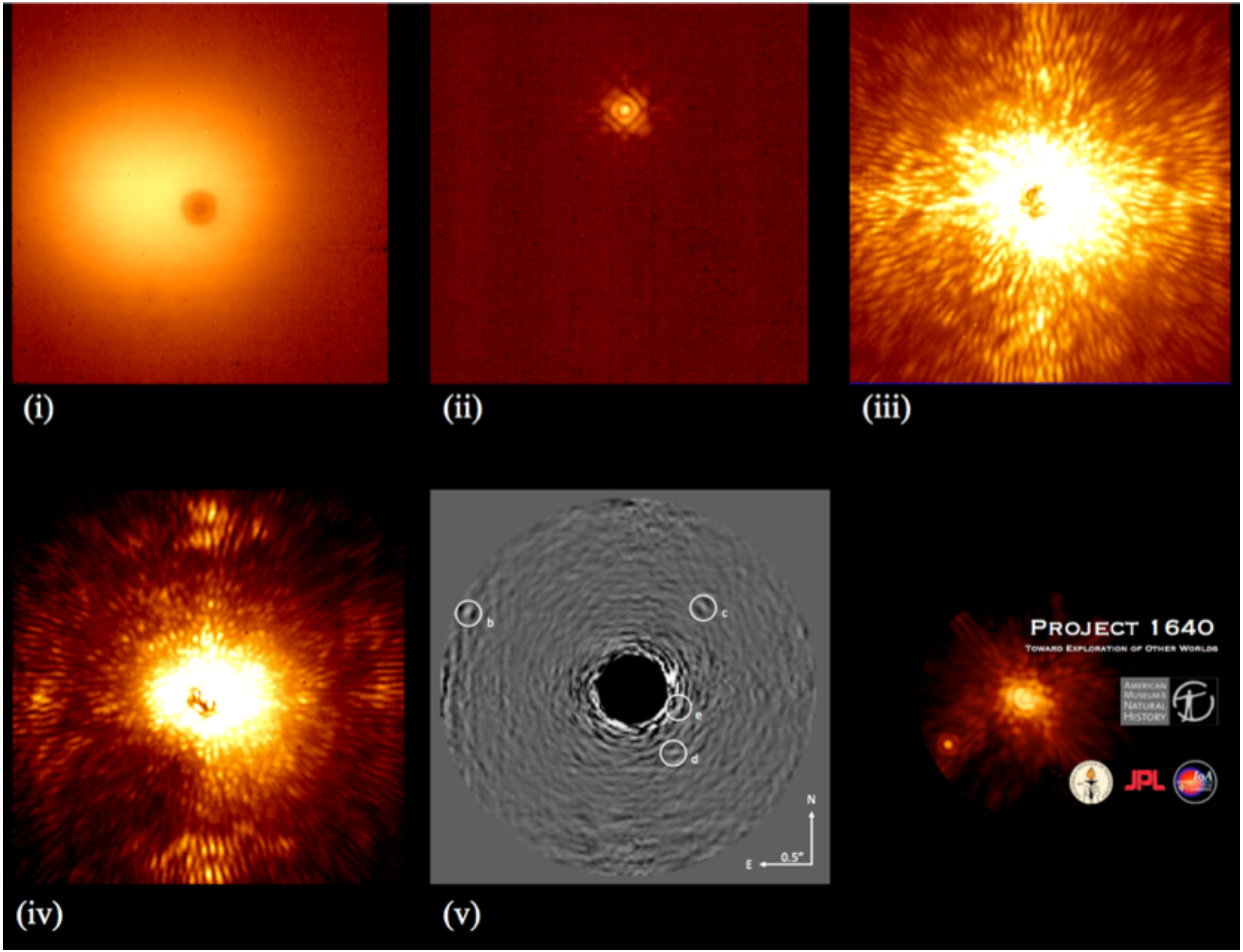
- **β** Pictoris b imaged with the AGPM on VLT/NACO
- Dataset used later for modelling and synthetic data generation

Example images





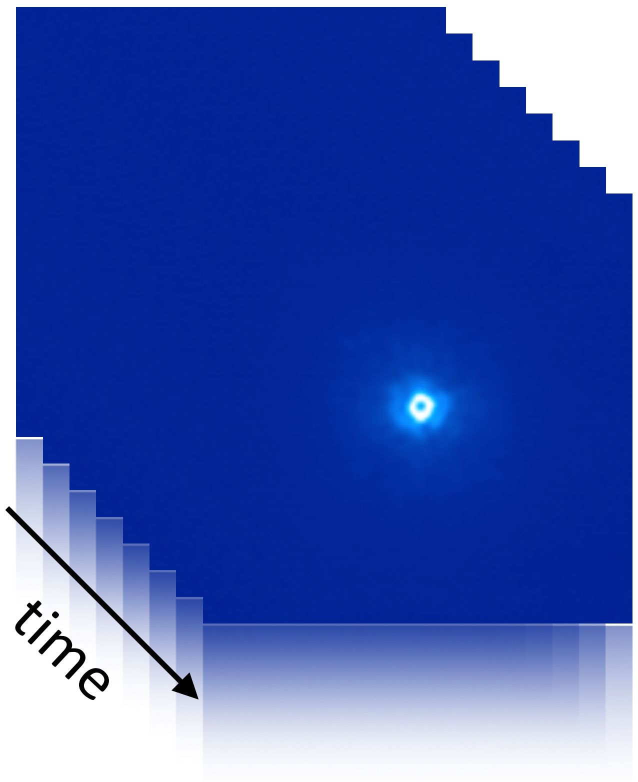
4 pillars of HiCI at small angles



Project 1640

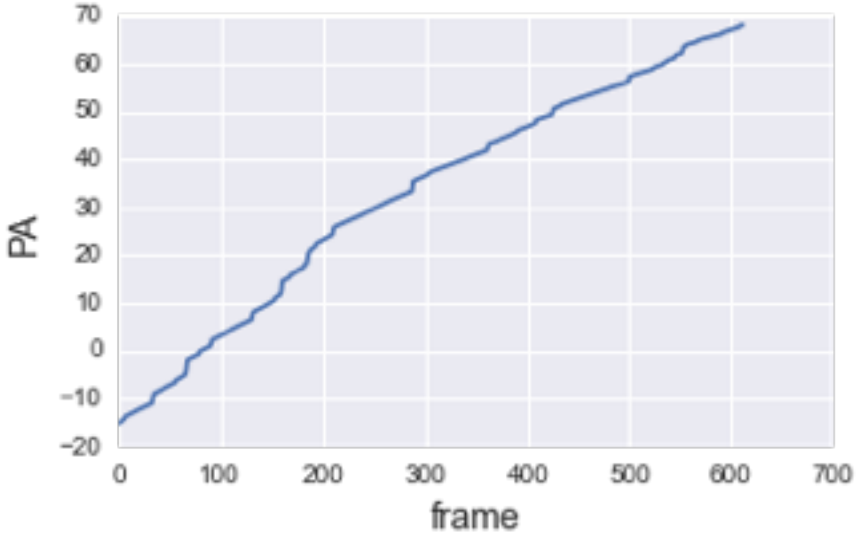
ADI Post-processing

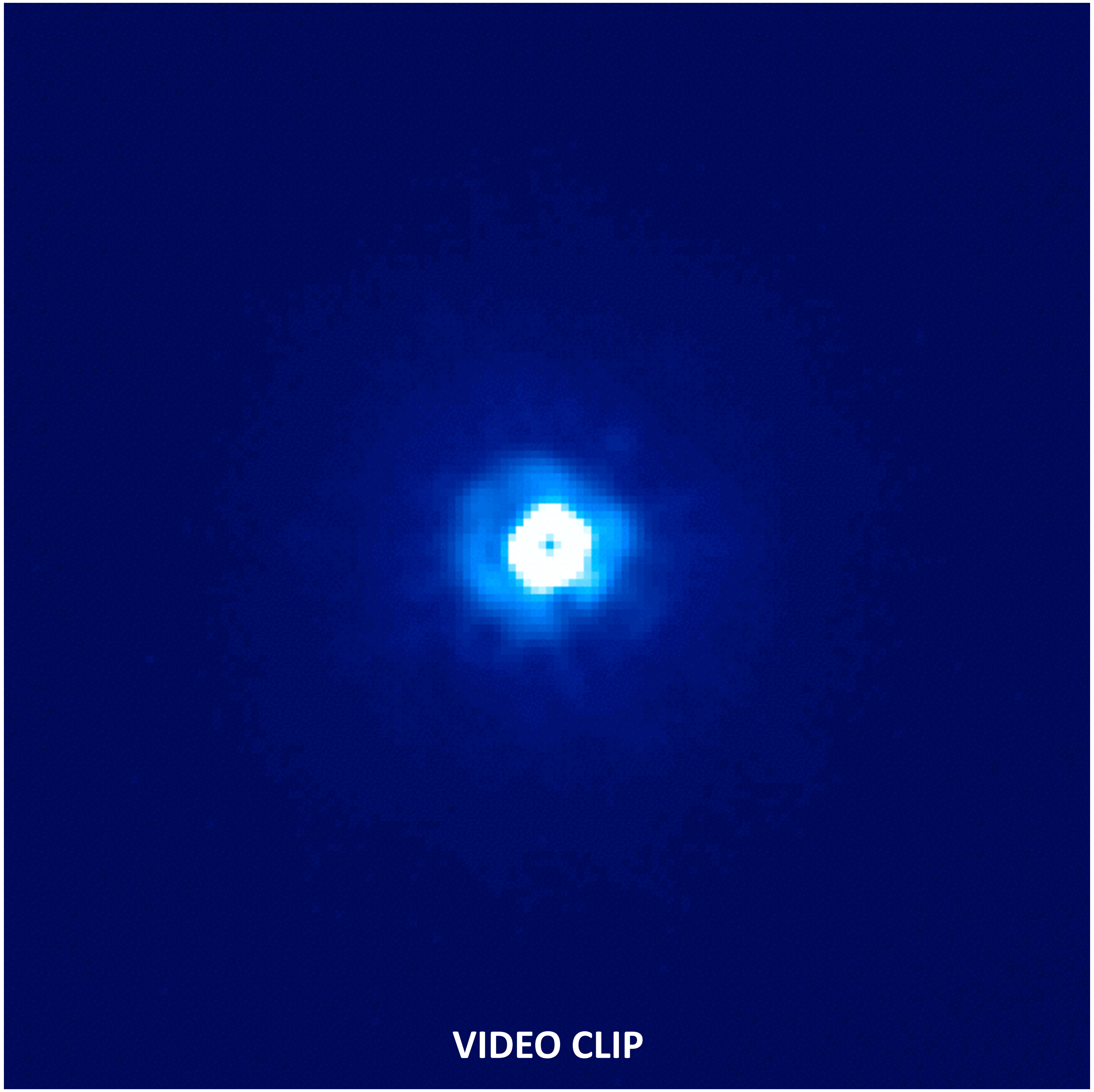
Image sequence of ***n*** calibrated images



n x ***p*** x ***p***

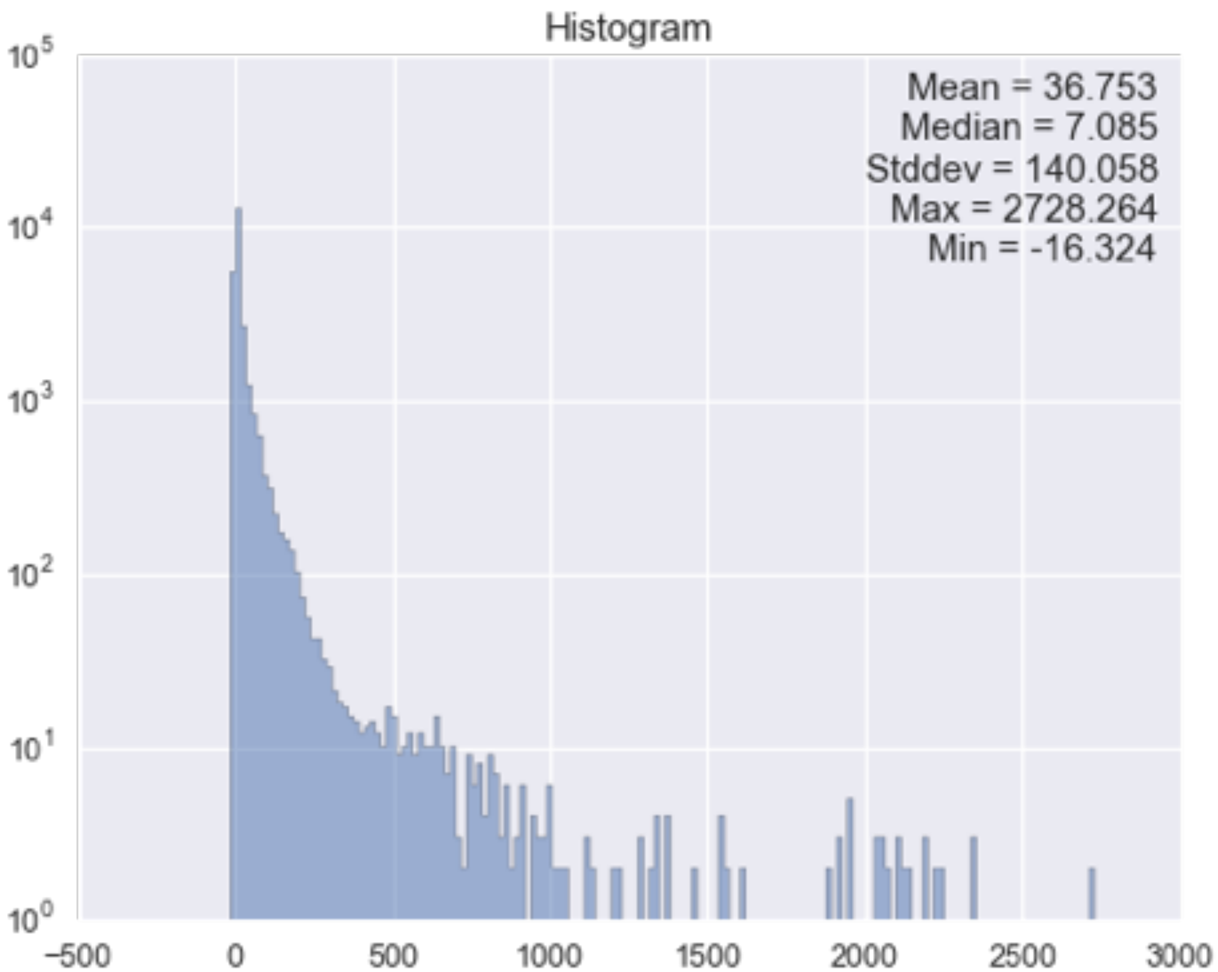
612 x 161 x 161

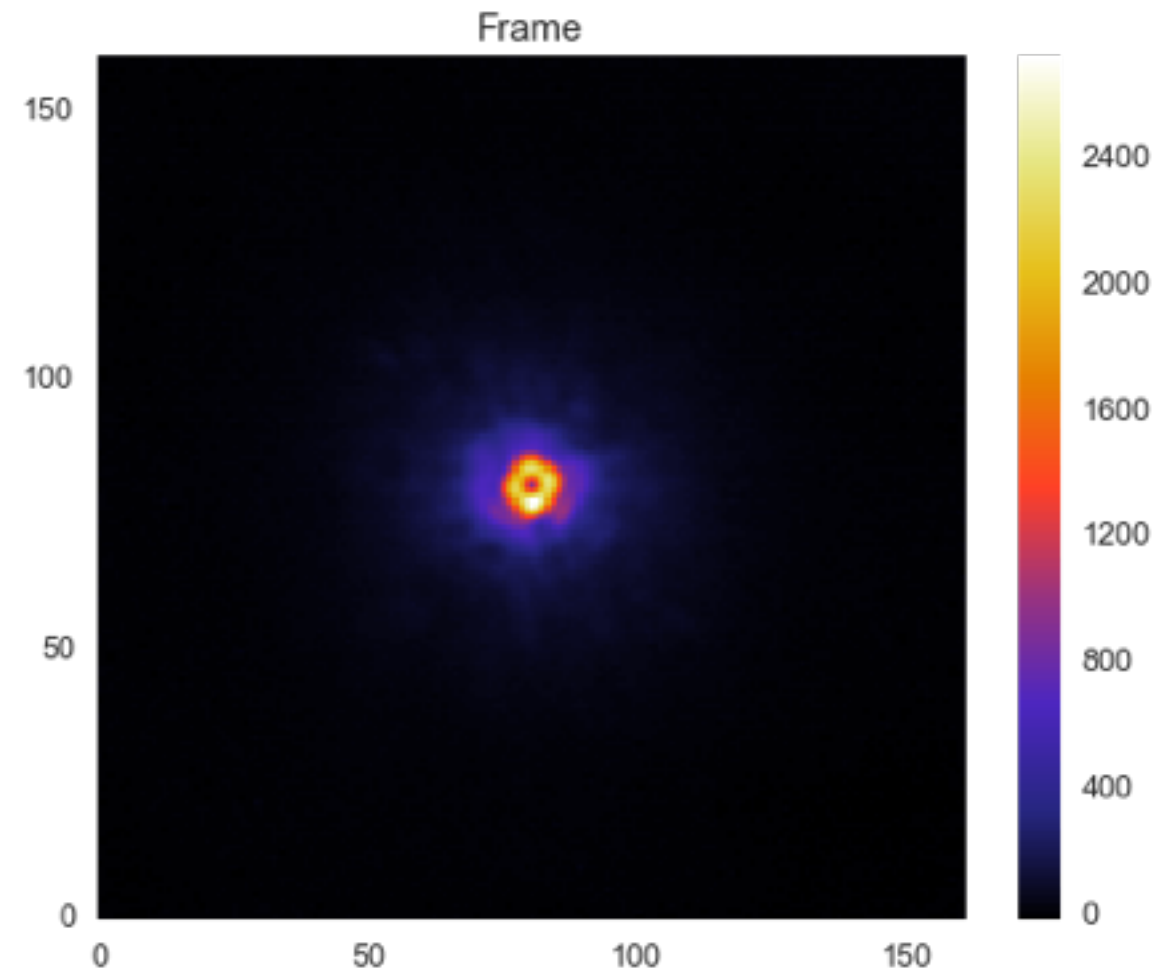




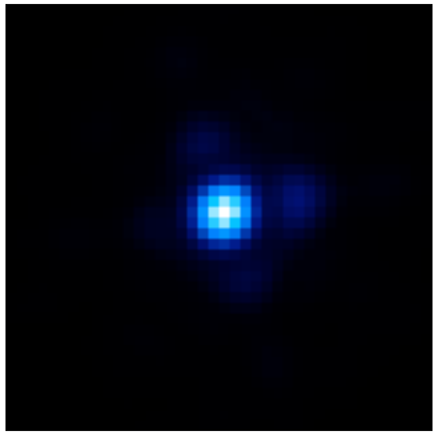
ADI Post-processing

Single frame



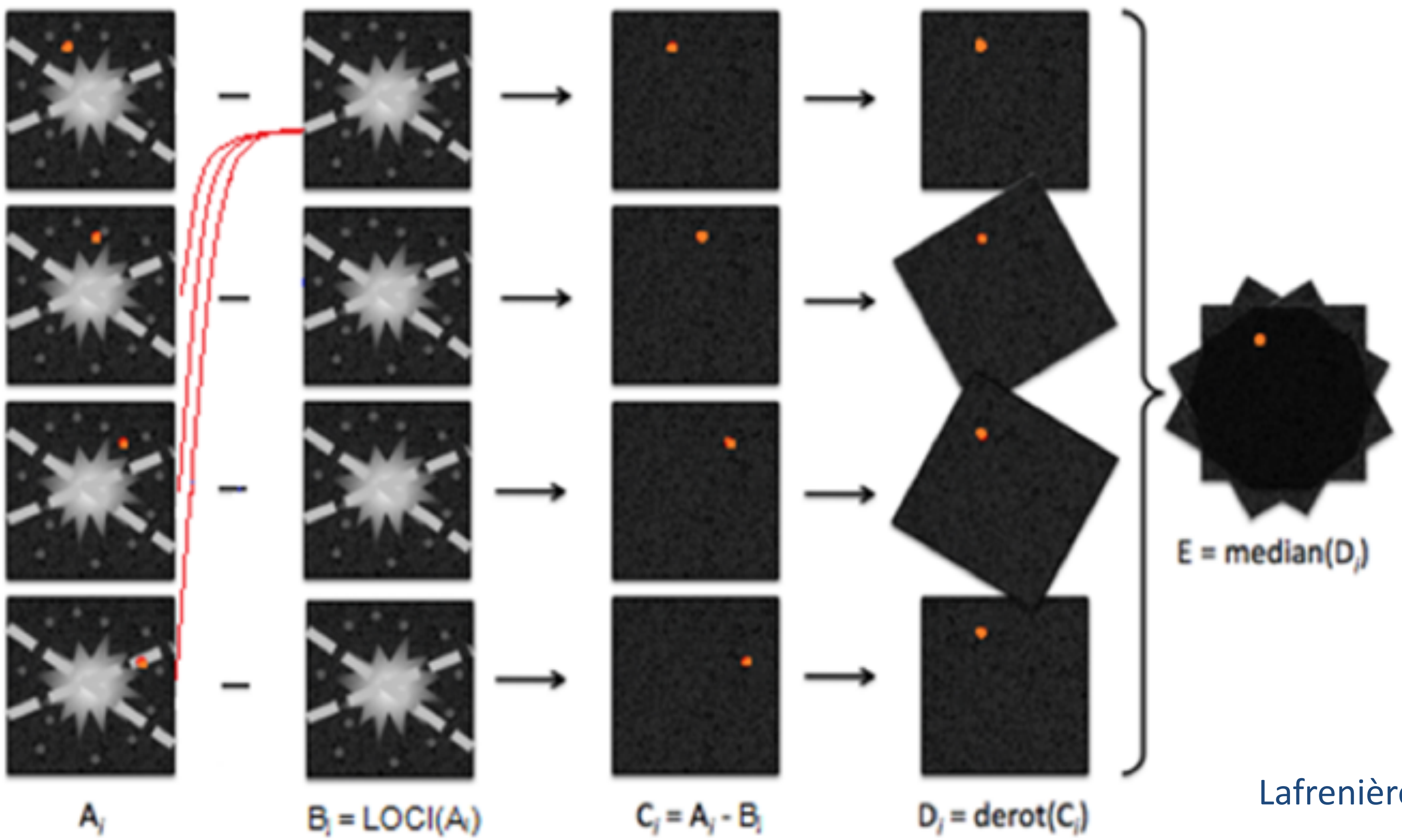


Off-axis Point Spread Function (**PSF)**

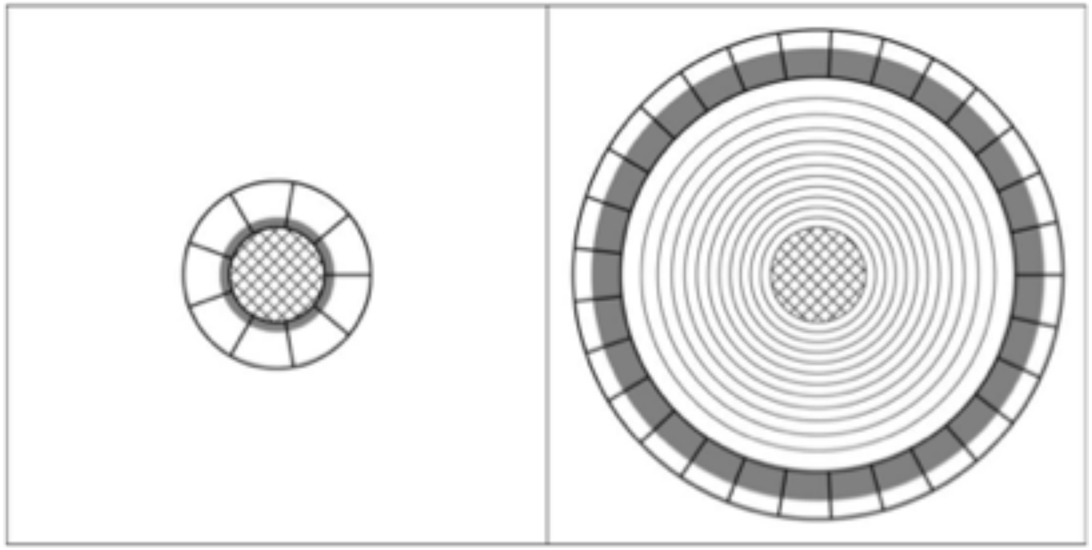


LOCI

Locally Optimized Combination of Images

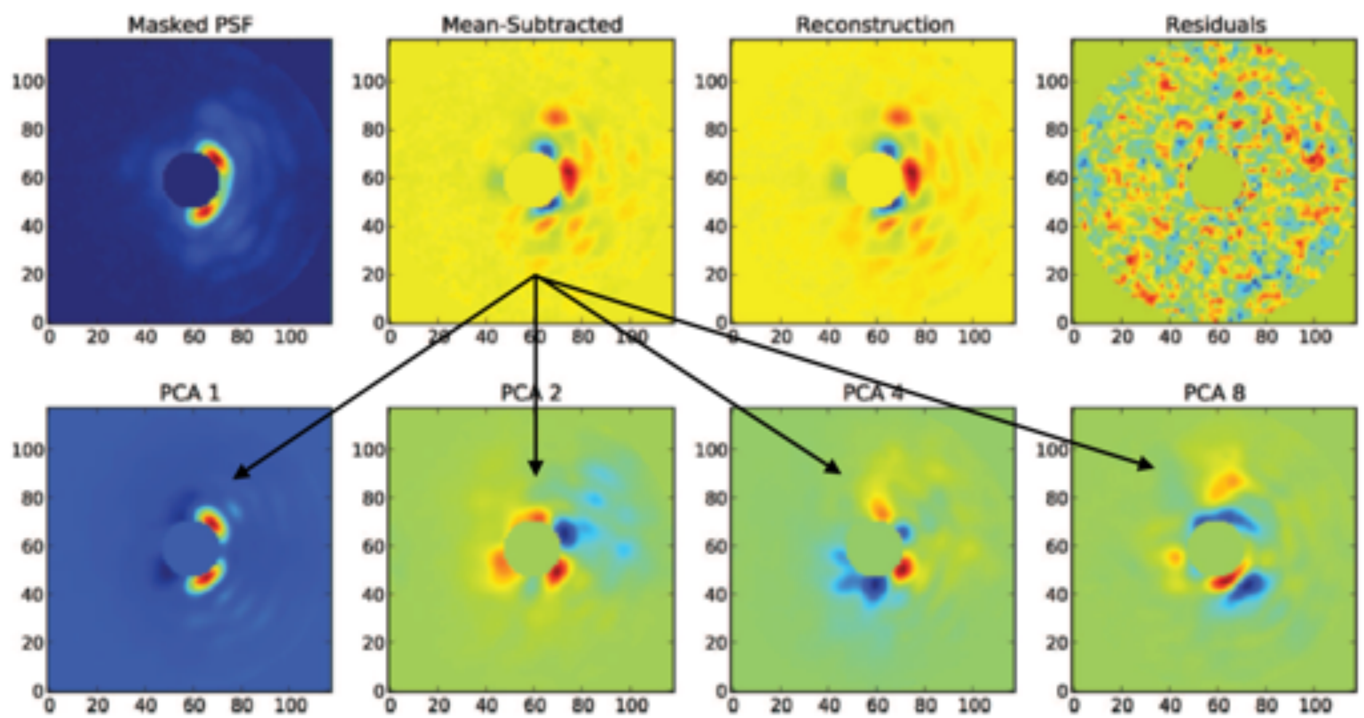


Lafrenière et al. 2007



- Works on small patches
- Computationally intensive
- Large number of free parameters

ADI-PCA



A. Amara and S. Quanz 2012

Let's consider a rectangular matrix of vectorized images:

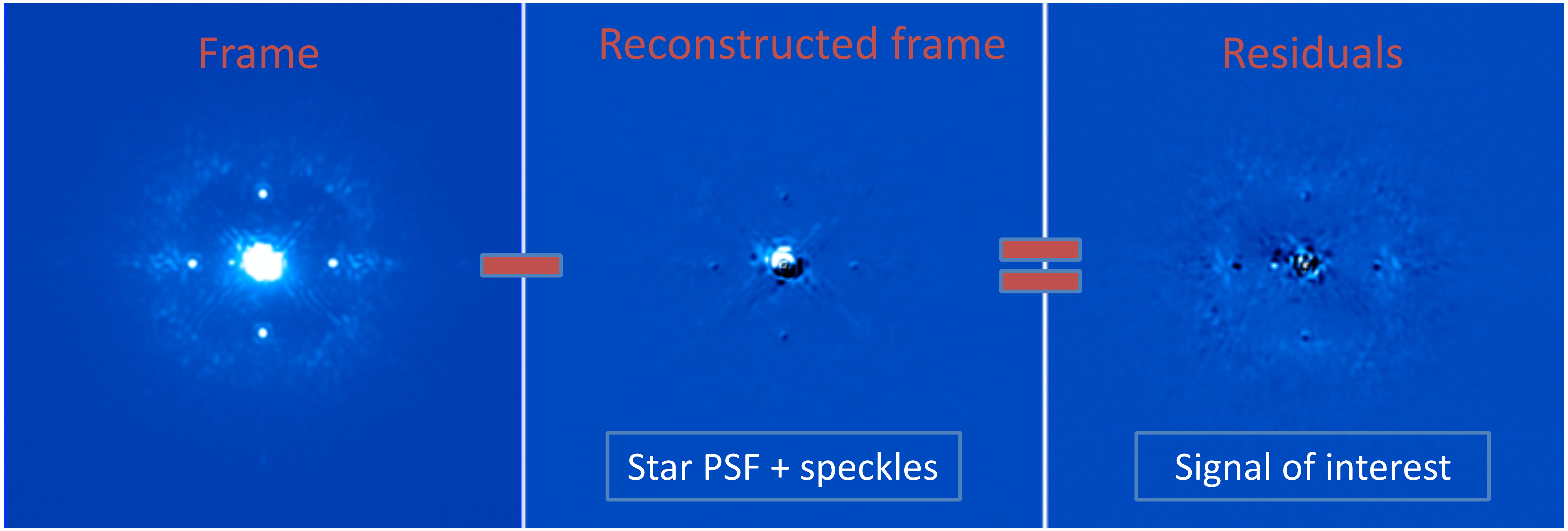
$$M \in \mathbb{R}^{m \times n}$$

Principal components can be computed via the eigenvectors of the covariance ***(MM^T)*** or singular value decomposition of ***M***

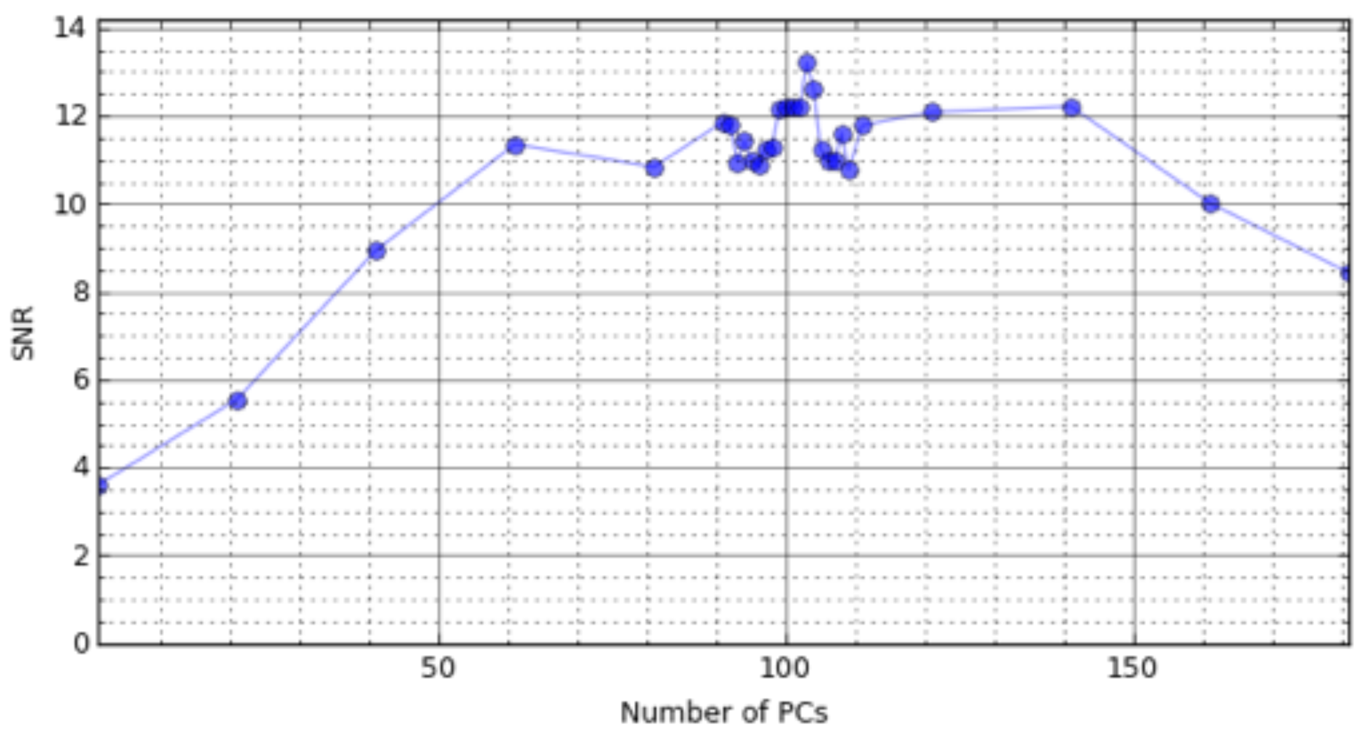
The reconstruction of the images using the first PCs gives a good model of the PSF +speckles

VORTEX

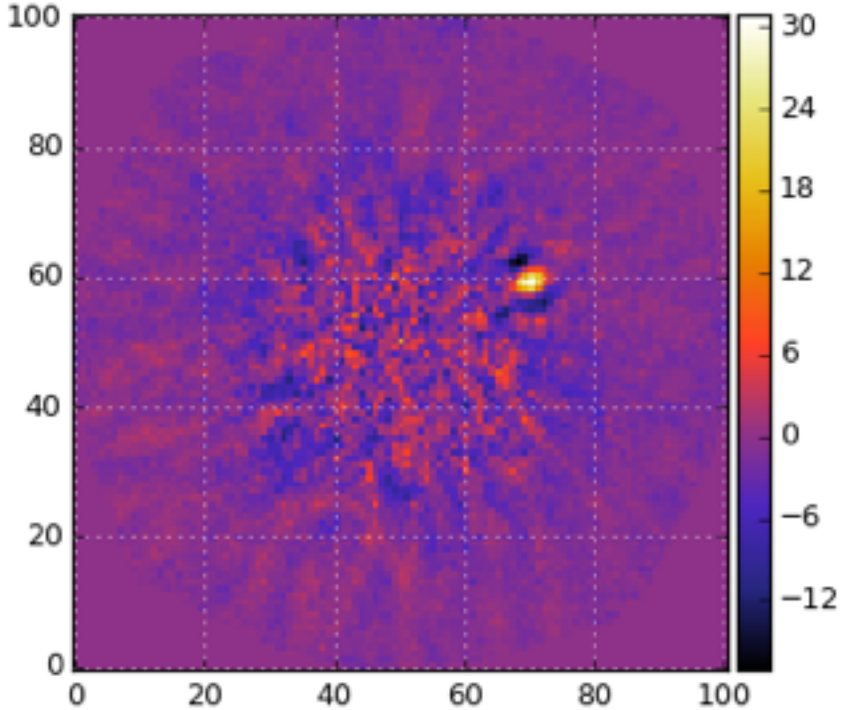
ADI-PCA



Frame
Reconstructed frame
Residuals
Star PSF + speckles
Signal of interest

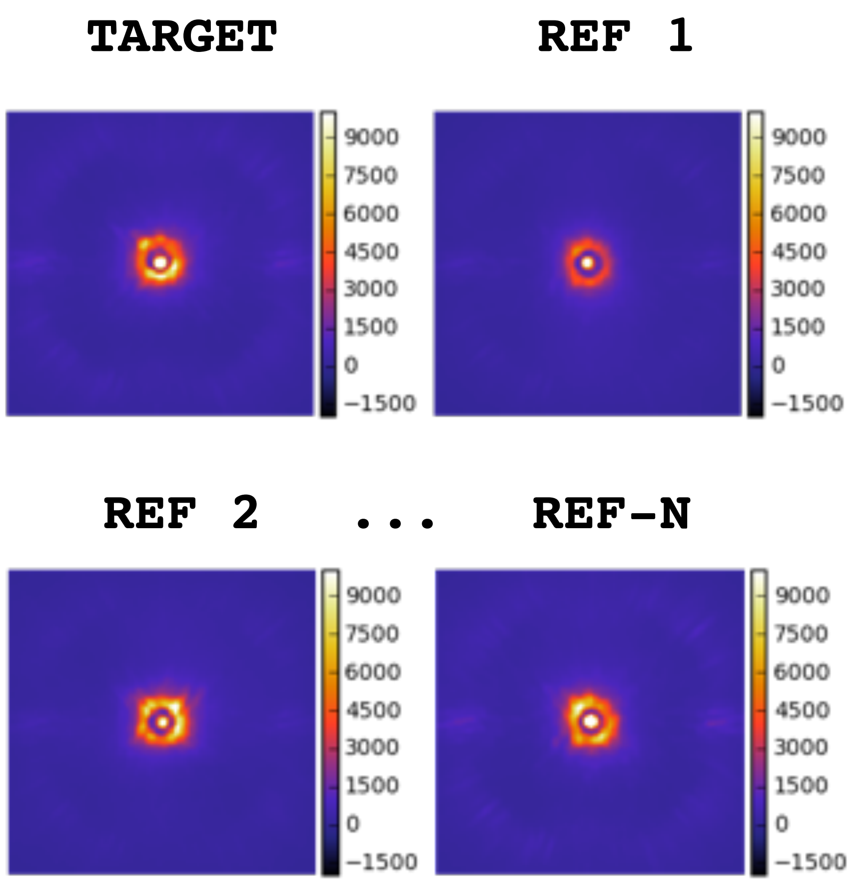


SNR
Number of PCs
0
2
4
6
8
10
12
14
50
100
150

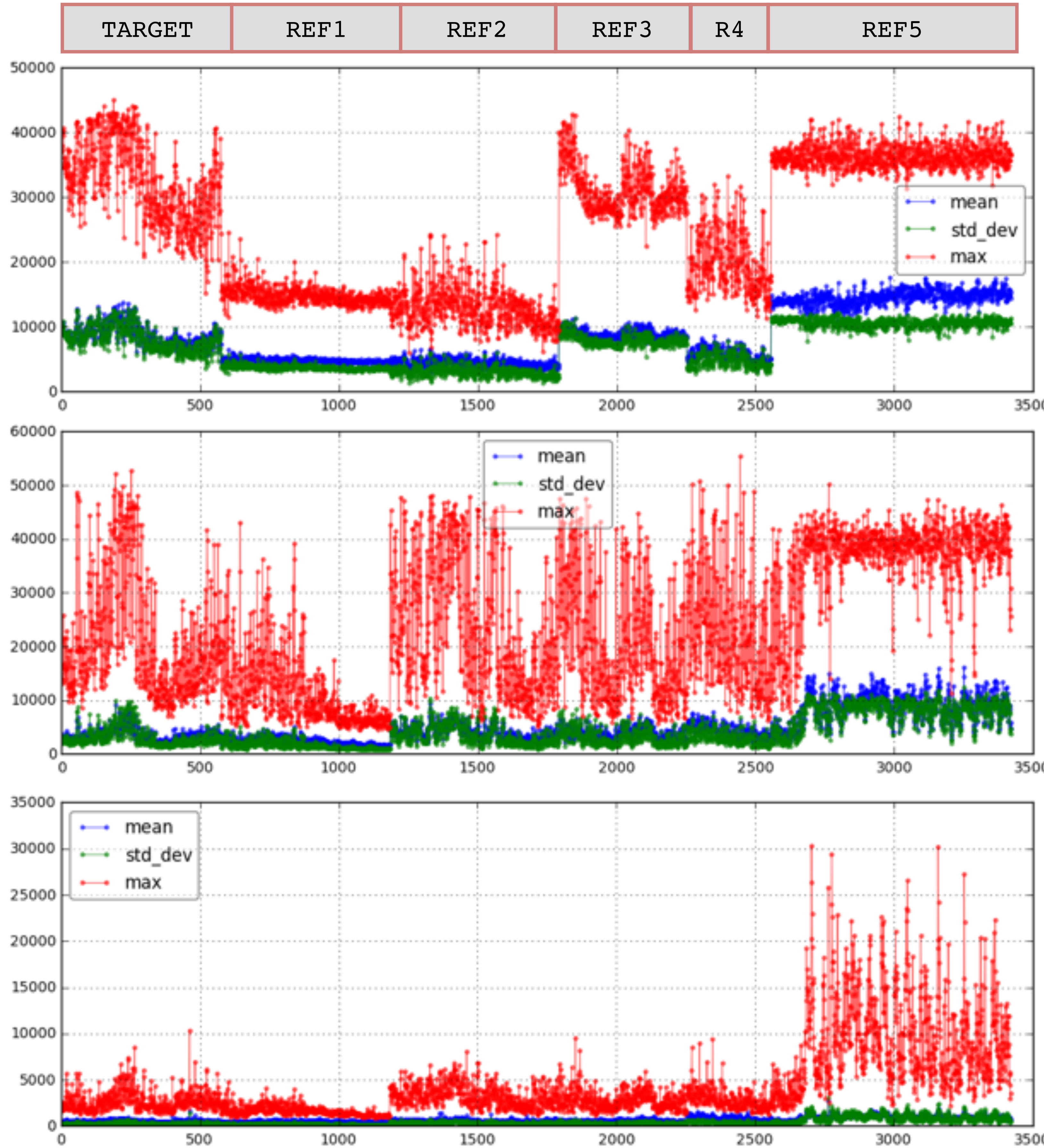


0
20
40
60
80
100
30
24
18
12
6
0
−6
−12

RDI-PCA



TARGET
REF 1
REF 2
...
REF–N
9000
7500
6000
4500
3000
1500
0
−1500



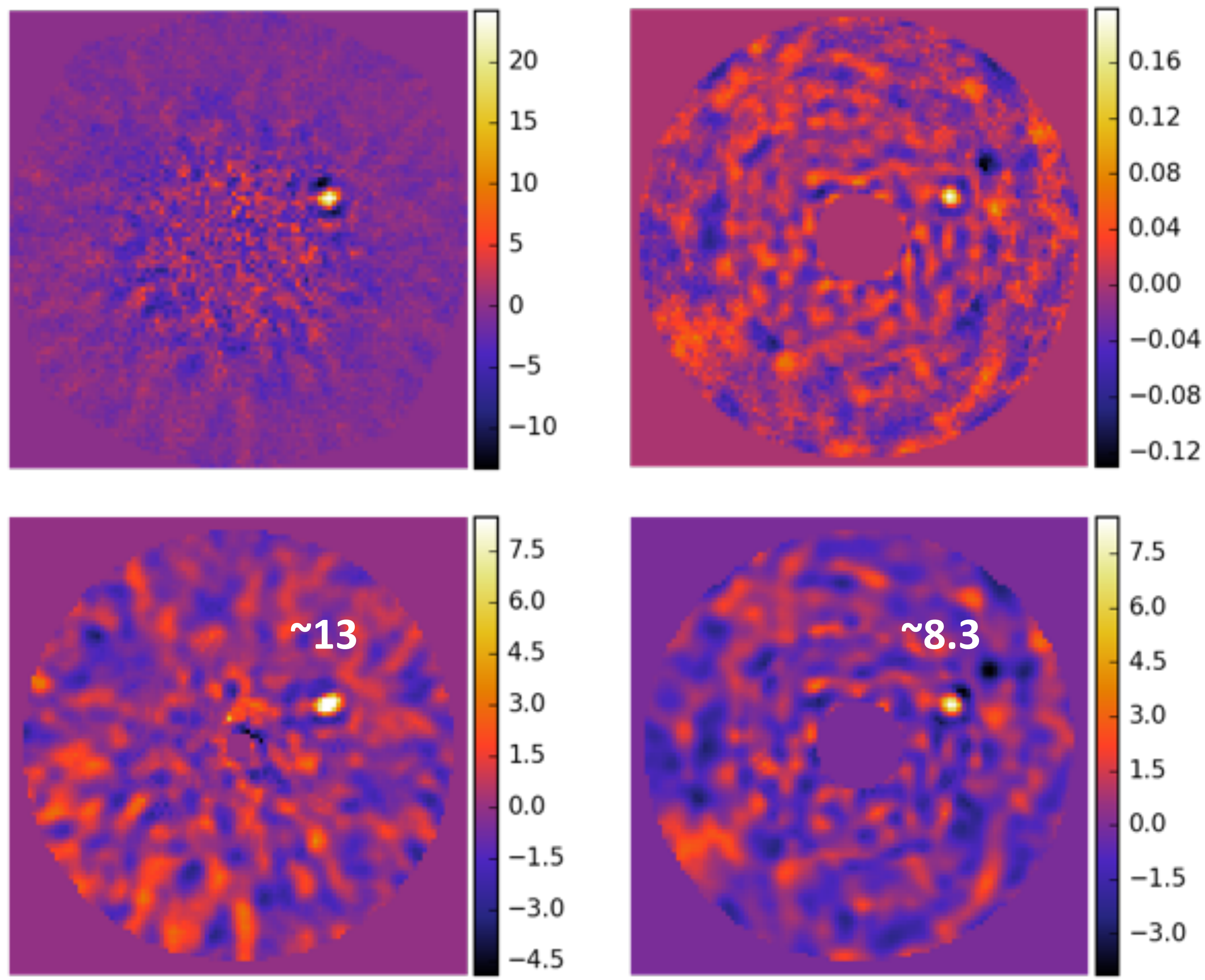
TARGET
REF1
REF2
REF3
R4
REF5
mean
std_dev
max

RDI-PCA

```
corr(median(TARGET), SUPEREF_i)
```

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

ADI-PCA RDI-PCA

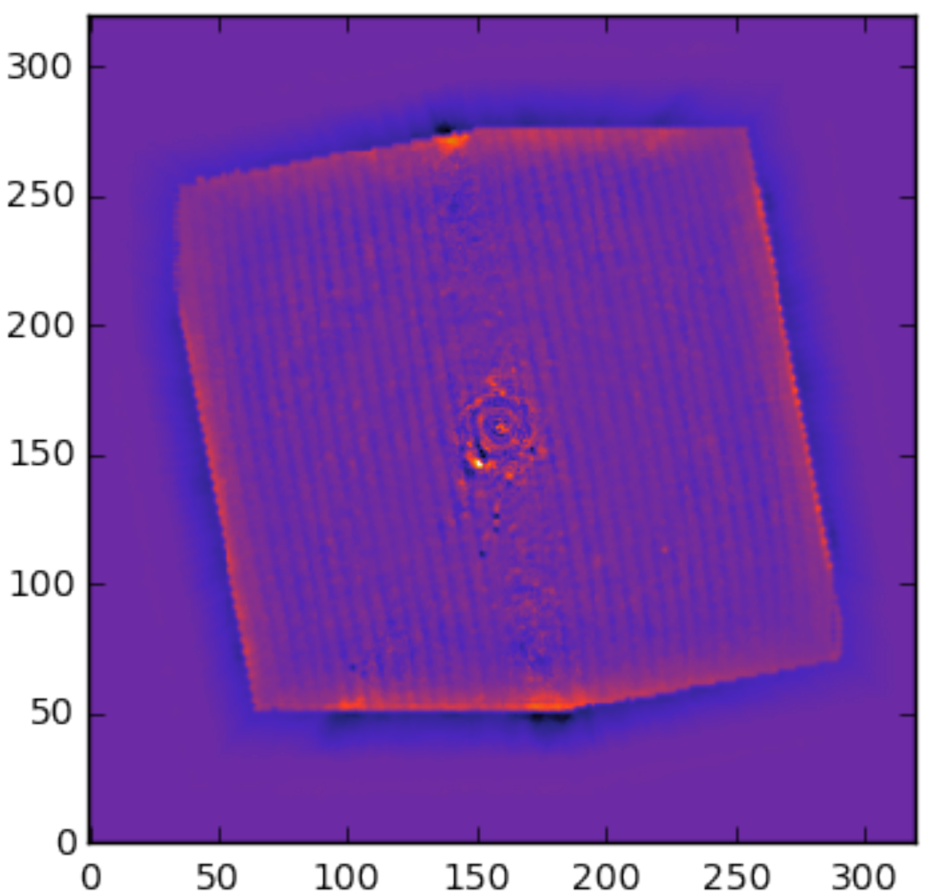


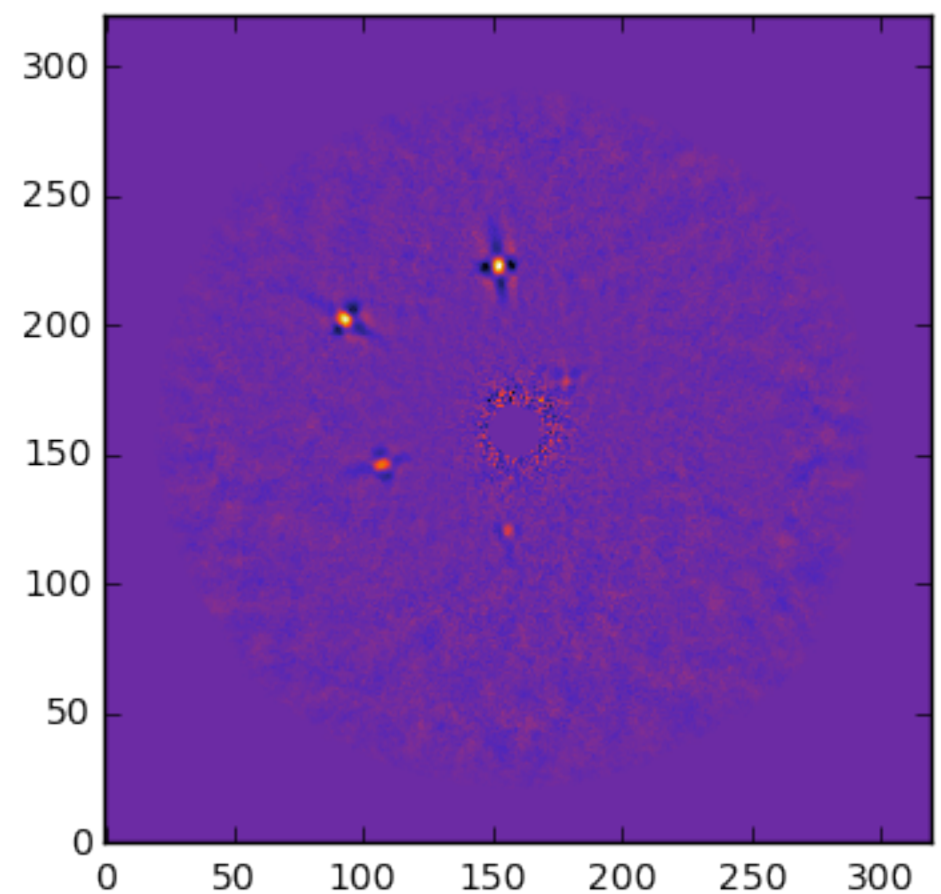
SDI-PCA (IFS)

w x ***p*** x ***p***
w - # λ

n x ***w*** x ***p*** x ***p***
n - # frames
w - # λ

Properly rescaling the ***w*** images, we'll have the planet moving radially. With ADI+IFS data we need to perform a two-stages PCA.





PCA

Implementations for high-contrast imaging: KLIP (Soummer et al. 2012), Pynpoint (Amara & Quanz 2012), VIP (Gomez Gonzalez et al. in prep). Some CONS:

- Self-subtraction reduced, but still present
- Fake companions injection still needed to retrieve astrometry and photometry

Connection with low-rank approximations:

$$M = L + E,$$

where ***L*** has low-rank and can be estimated by the best low-rank approximation of ***M*** in the least-scare sense:

$$\min_{L} ||M - L||_F^2, \text{ subject to } \text{rank}(L) \leq k,$$

where ***k*** is the rank of the low-rank approximation ***L***

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PCA

According to the matrix approximation lemma (Eckart & Young 1936), the previous problem can be solver through SVD:

$$M = U\Sigma V^T = \sum_{i=1}^{k} \sigma_i u_i v_i^T,$$

where the vectors $\boldsymbol{u_i}$ and $\boldsymbol{v_i}$ are the left and right singular vectors and $\boldsymbol{\sigma_i}$ the singular values of ***M***. Choosing the first ***k*** left singular vectors forms an orthonormal basis that captures most of the variance of ***M***. This corresponds to PCA.

- PCA was used before in computer vision for background subtraction (Oliver et al 2000)
- Indeed this are very similar problems
- For background subtraction other subspace projection algorithms have been proposed recently

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Robust PCA

RPCA algorithms models the data as the superposition of low-rank and sparse components. One of the first approaches, PCP (Candès et al. 2009):

$$\min_{L,S} \|L\|_* + \lambda \|S\|_1 \text{ , subject to } L + S = M,$$

where ***L*** is low-rank and ***S*** contains sparse signal of arbitrarily large magnitude.

This doesn't work as well for ADI image sequences. They can be decomposed exactly in low-rank and sparse components. The quasi-static speckles still pollute ***S*** and the planet signal is spatially correlated.

We opt for a three-term decomposition, including a noise term.

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L+S+G

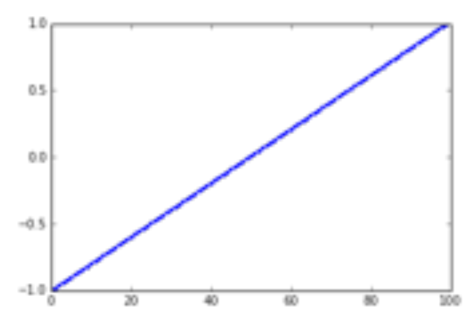
GoDec/SSGoDec (Zhou 2011, Zhou & Tao 2013). Fast implementation of a low rank plus sparse decomposition. Different formulation:

$$M = L + S + G, \mathrm{rank}(L) \le k, \mathrm{card}(S) \le c,$$

where ***G*** is a dense noise term. In terms of the decomposition error:

$$\min_{L,S} \|M - L - S\|_F^2, \text{ subject to } \mathrm{rank}(L) \le k, \ \mathrm{card}(S) \le c.$$

$$\begin{cases} L_t = \underset{\mathrm{rank}(L)\le k}{\operatorname{argmin}} \|M - L - S_{t-1}\|_F^2; \\ S_t = \underset{\mathrm{card}(S)\le c}{\operatorname{argmin}} \|M - L_t - S\|_F^2. \end{cases}$$

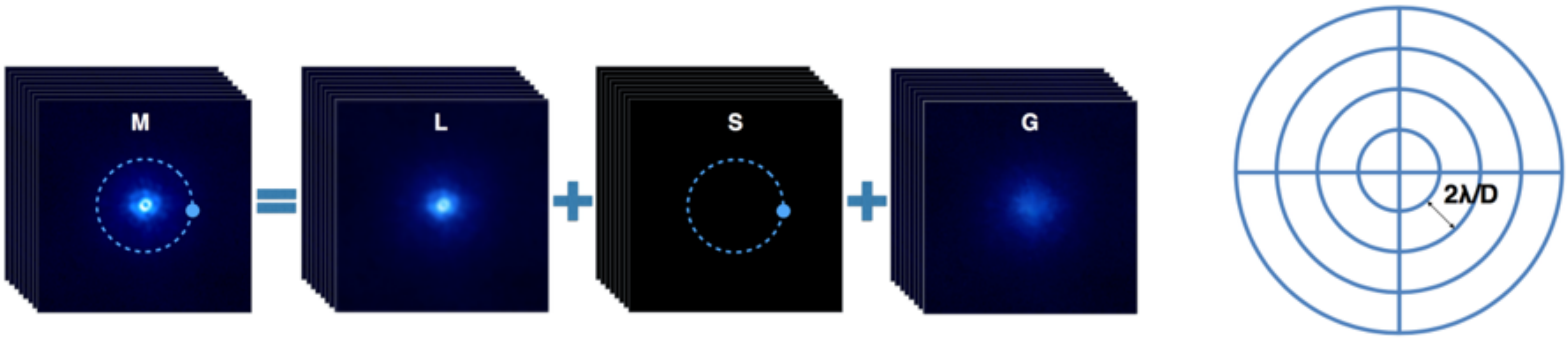


$\boldsymbol{L_t}$ can be updated via the Bilateral Random Projections (Zhou & Tao 2013) of $\boldsymbol{M\text{-}S_{t-1}}$ and $\boldsymbol{S_t}$ via entry-wise soft-thresholding of $\boldsymbol{M\text{-}L_t}$.

$$S_\gamma X = \mathrm{sgn}(X_{ij}) \max\left(|X_{ij}| - \gamma, 0\right).$$

LLSG

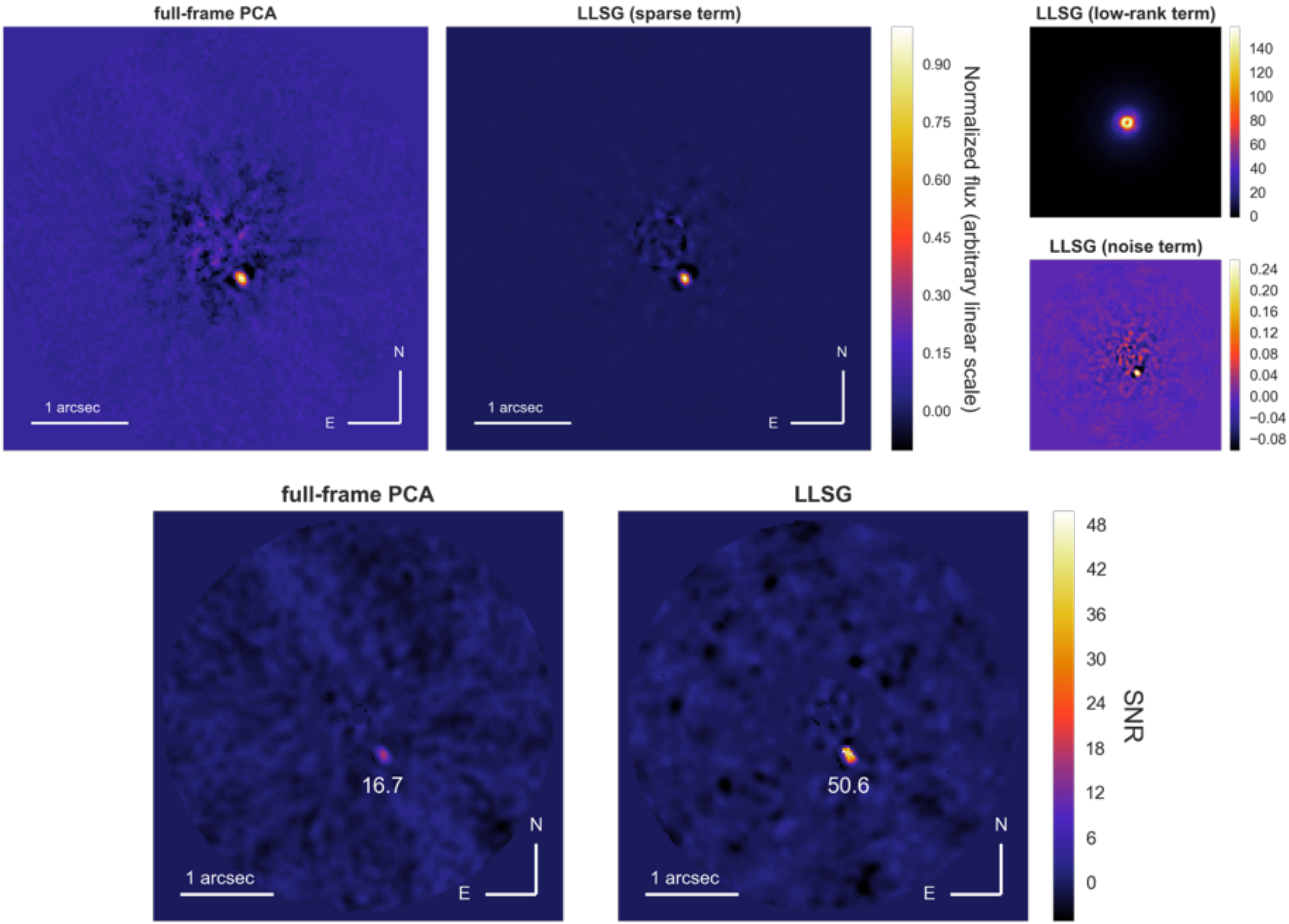
Local Low-rank plus Sparse plus Gaussian noise decomposition



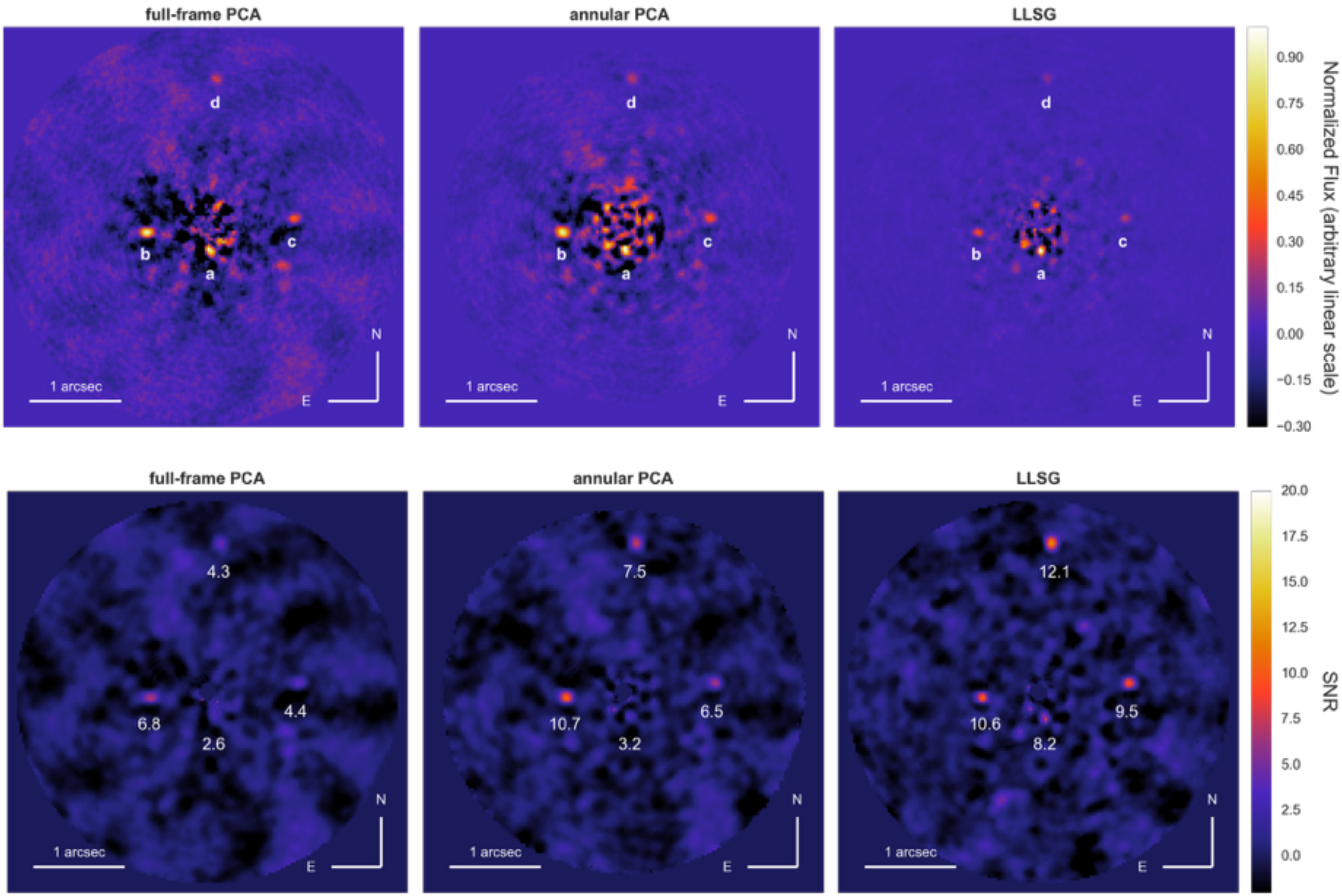
Local three-term decomposition of ADI cubes (Gomez Gonzalez et al., in press). The goal is to boost the detection of point-like sources. Steps:

- images are broken into patches,
- each quadrant is decomposed separately alternating BRP and soft-thresholding for a fixed number of iterations,
- for each patch, ***S*** is kept,
- all frames are rotated to a common north and median combined.

Application to real data

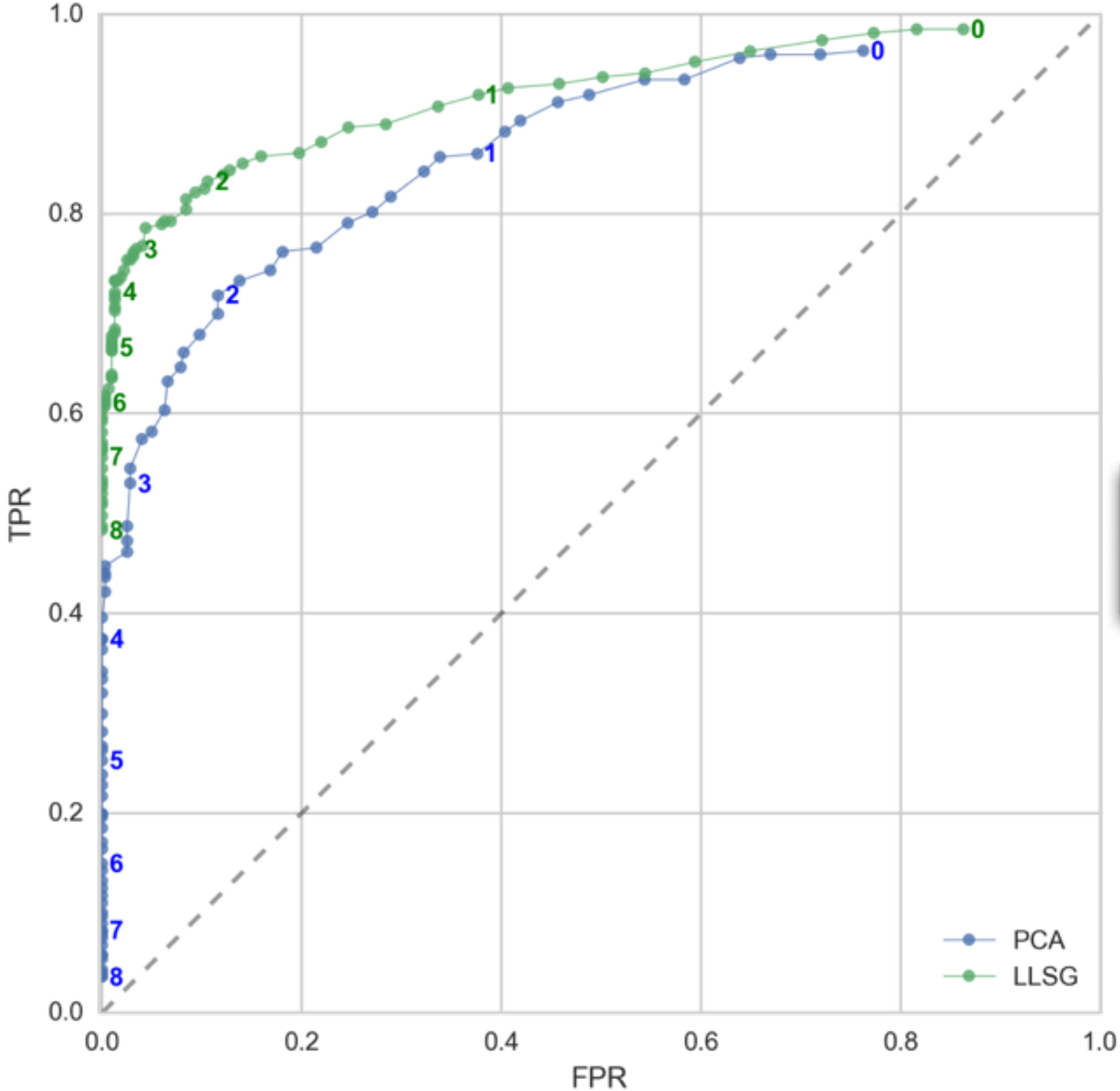


Synthetic companions



VORTEX

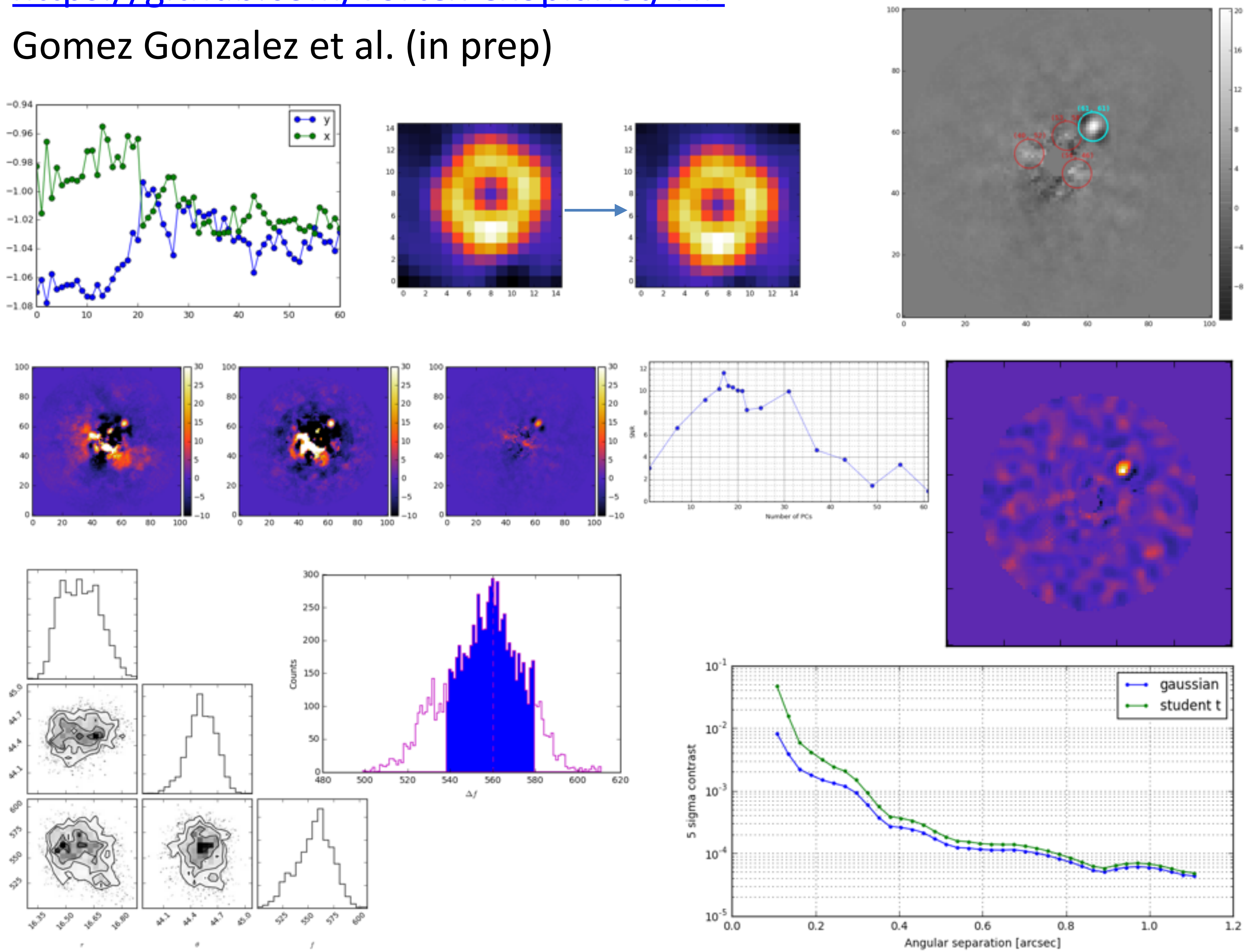
ROC performance



	H_1: signal present	H_0: signal absent
Detection	True Positive	False Positive *type I error*
Null result	False Negative *type II error*	True Negative
	TPF = sensitivity = TP/(TP+FN)	FPF = 1-specificity = 1-CL = FP/(FP+TN)

https://github.com/vortex-exoplanet/VIP

Gomez Gonzalez et al. (in prep)



Takeaway points

- Direct imaging of exoplanets is a very difficult enterprise (future)
- Post-processing plays a critical role
- PCA works well, LLSG can boost SNR, especially at small angles.
- LLSG has a computational cost comparable to the one of PCA. Parallelism can be exploited
- Astrophysicist need to work together with machine learning/image processing community

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Thank you!